The Ballot

Charles M. Biles
nia977.wixsite.com/drbcap

College of the Redwoods
POLSC 10
8 November 2016
Election Day!

1

Advice

Charles M. Biles
nia977.wixsite.com/drbcap

College of the Redwoods
POLSC 10
8 November 2016
Election Day!

Advice

Charles M. Biles
nia977.wixsite.com/drbcap

College of the Redwoods
POLSC 10
8 November 2016
Election Day!

Advice

Charles M. Biles   College of the Redwoods
nia977.wixsite.com/drbcap   POLSC 10
                                        8 November 2016
                                          Election Day!

4

Question

Have you been following the presidential election?

Question

What is an election?

Question

What is the difference between
a war and an election?

Question

A prime goal of democracy is to replace wars with elections by replacing bullets with ballots.

The Ballot

An election must feature a ballot. We will assume the ballot is the same for each voter; further, one person/one ballot.

The Ballot

The structure of the ballot determines your voice in an election.

The Ballot

During the Stalin era of the Soviet Union, a ballot looked like this:

The Ballot

During the Stalin era of the Soviet Union, a ballot looked like this:

District Commisar
Vote for one:

❑ Alesander Kolnovic

The Ballot

A two-option ballot looks like this:

Vote for One

❑ Option A

❑ Option B

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

➢ Majority Rule

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

➢ Majority Rule

➢ Unanimous

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

➢ Majority Rule

➢ Unanimous

➢ Super majority

The Single Vote Ballot

A multi-option ballot looks like this:

Vote for One

❑ Option A
❑ Option B
❑ Option C
❑ Option D
❑ Option E

The Single Vote Ballot

A multi-option ballot looks like this:

Vote for One

☐ Option A
☐ Option B
☐ Option C
☐ Option D
☐ Option E

Who wins?

The Single Vote Ballot

A multi-option ballot looks like this:

> **Vote for One**
>
> ❑ Option A
> ❑ Option B
> ❑ Option C
> ❑ Option D
> ❑ Option E

Who wins?   Plurality

The Single Vote Ballot

A multi-option ballot looks like this:

> **Vote for One**
>
> ❑ Option A
> ❑ Option B
> ❑ Option C
> ❑ Option D
> ❑ Option E

Who wins?   Plurality

Majority else top 2 run-off

The Single Vote Ballot

A multi-option ballot looks like this:

Vote for One

❑ Option A
❑ Option B
❑ Option C
❑ Option D
❑ Option E

Notice how restricted the voter's voice is.

Who wins?   Plurality
                  Majority else top 2 run-off

The Single Vote Ballot

A multi-option ballot looks like this:

Vote for One

❑ Option A
❑ Option B
❑ Option C
❑ Option D
❑ Option E

Notice how restricted the voter's voice is.

Many voters will vote strategically rather than honestly.

Who wins?   Plurality
                 Majority else top 2 run-off

Approval Voting

A multi-option ballot looks like this:

Approval List

❑ Option A
❑ Option B
❑ Option C
❑ Option D
❑ Option E

Vote for all options that you approve.

Approval Voting

A multi-option ballot looks like this:

Approval List

- ❏ Option A
- ❏ Option B
- ❏ Option C
- ❏ Option D
- ❏ Option E

Vote for all options that you approve.

Who wins?

- ➤ Plurality
- ➤ Top 2 run-off if no majority
- ➤ Greatest majority

Approval Voting

A comparison. Here, $N = 100$.

Consider 100 voters in a three-way election:

- ➢ 26 first choice A but approve of B.
- ➢ 25 first choice A but approve neither B nor C.
- ➢ 15 first choice B but approve neither A nor C.
- ➢ 18 first choice C but approve of B.
- ➢ 16 first choice C but approve neither A nor B.

Approval Voting

A comparison. Here, N = 100.

Consider 100 voters in a three-way election:

- ➢ 26 first choice A
- ➢ 25 first choice A
- ➢ 15 first choice B
- ➢ 18 first choice C
- ➢ 16 first choice C

In a Vote for One election, A wins by majority rule.

Approval Voting

A comparison. Here, N = 100.

26	25	15	18	16
● A	● A	○ A	○ A	○ A
● B	○ B	● B	● B	○ B
○ C	○ C	○ C	● C	● C

vs. Vote for One option.

51	15	34
● A	○ A	○ A
○ B	● B	○ B
○ C	○ C	● C

Approval Voting

A comparison. Here, *N* = 100.

26	25	15	18	16
● A	● A	○ A	○ A	○ A
● B	○ B	● B	● B	○ B
○ C	○ C	○ C	● C	● C

A 51
B 59
C 34
B wins.

vs. Vote for One option.

51	15	34
● A	○ A	○ A
○ B	● B	○ B
○ C	○ C	● C

A 51
B 15
C 34
A wins.

Which Ballot?

Vote for One Party

❑ **Hillary Clinton**
    and Tim Kaine
   Democratic

❑ **Gloria Estela La Riva**
    and Dennis J. Banks
   Peace and Freedom

❑ **Donald J. Trump**
    and Michael R. Pence
   Republican, American Independent

❑ **Gary Johnson**
    and Bill Weld
   Libertarian

❑ **Jill Stein**
    and Ajamu Baraka
   Green

Which Ballot?

Vote for One Party

❑ Hillary Clinton
   and Tim Kaine
Democratic

❑ Gloria Estela La Riva
   and Dennis J. Banks
Peace and Freedom

❑ Donald J. Trump
   and Michael R. Pence
Republican, American Independent

❑ Gary Johnson
   and Bill Weld
Libertarian

❑ Jill Stein
   and Ajamu Baraka
Green

Vote for all Approved

❑ Hillary Clinton
   and Tim Kaine
Democratic

❑ Gloria Estela La Riva
   and Dennis J. Banks
Peace and Freedom

❑ Donald J. Trump
   and Michael R. Pence
Republican, American Independent

❑ Gary Johnson
   and Bill Weld
Libertarian

❑ Jill Stein
   and Ajamu Baraka
Green

Ranked Choice Voting

In a ranked choice ballot the voter ranks some or all of the candidates.

In a top three system, you rank your top 3 choices as 1, 2, or 3.

In a full ranked system, if the ballot displays five choices, then you rank those choices 1 though 5.

The Ballot

In the 2010 Senate Race in California, consider these two ballots.

Vote for one:
- Duane Roberts (G)
- Marsha Feinland (PF)
- Gail Lightfoot (L)
- Barbara Boxer (D)
- Carly Fiorina (R)
- Edward Noonan (AI)

G = Green
PF = Peace & Freedom
L = Libertarian
AI = American Independent

The Ballot

In the 2010 Senate Race in California, consider these two ballots.

Vote for one:

- o   Duane Roberts (G)
- o   Marsha Feinland (PF)
- o   Gail Lightfoot (L)
- o   Barbara Boxer (D)
- o   Carly Fiorina (R)
- o   Edward Noonan (AI)

G = Green
PF = Peace & Freedom
L = Libertarian
AI = American Independent

Rank each candidate once, 1 – 6:

RANK	1	2	3	4	5	6
Duane Roberts	o	o	o	o	o	o
Marsha Feinland	o	o	o	o	o	o
Gail Lightfoot	o	o	o	o	o	o
Barbara Boxer	o	o	o	o	o	o
Carly Fiorina	o	o	o	o	o	o
Edward Noonan	o	o	o	o	o	o

The Ballot

In the 2010 Senate Race in California, consider these two ballots.

Vote for one:

- o Duane Roberts (G)
- o Marsha Feinland (PF)
- ● Gail Lightfoot (L)
- o Barbara Boxer (D)
- o Carly Fiorina (R)
- o Edward Noonan (AI)

G = Green
PF = Peace & Freedom
L = Libertarian
AI = American Independent

Rank each candidate once, 1 – 6:

RANK	1	2	3	4	5	6
Duane Roberts	o	●	o	o	o	o
Marsha Feinland	o	o	o	●	o	o
Gail Lightfoot	●	o	o	o	o	o
Barbara Boxer	o	o	●	o	o	o
Carly Fiorina	o	o	o	o	●	o
Edward Noonan	o	o	o	o	o	●

35

A Background Story

XYZ Company employs 55 people. Health insurance legislation requires XYZ to provide insurance to its employees. XYZ receives bids from five companies: A, B, C, D and E. XYZ must adopt one of these plans which then applies to all its employees. All five plans are the same cost to XYZ; however, the benefits package differs between plans. Previously the CEO of XYZ sent a letter to its employees that XYZ would allow them to vote among the competing bids and that XYZ would honor their choice. XYZ holds an election and the employees vote their preferences.

The Ballots:
55 voters

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

The Ballots:
55 voters

	18	12	10	9	4	2
First	A	B	C	D		E
Second	D	E			B	C
Third	E				D	D
Fourth		C		B	C	B
Fifth	B			A	A	A

Who's the winner?

Majority Criterion

The Majority Criterion is considered to be a basic rule of fairness:

Majority Criterion

The Majority Criterion is considered to be a basic rule of fairness:

If a candidate gets a majority of votes, then that candidate should be declared the winner.

40

The Problem

If no candidate gets a majority of the votes, then is there such a thing as the "will of the people?"

The Problem

If no candidate gets a majority of the votes, then is there such a thing as the "will of the people?"

In general, can individual preferences be translated into a "group choice?"

Decision Mechanisms

Decision Mechanisms

- Plurality

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

The candidate with the most first place votes wins.

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E

The candidate with the most first place votes wins.

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E

The candidate with the most first place votes wins.

A wins.

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N = 55

Plurality is the most common method in US elections.

Main drawback: the least desirable candidate may win. Here, the plurality winner A would lose 37-18 in a one-on-one contest with any other candidate.

Decision Mechanisms

- Plurality

- Top two run-off

Top Two Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E

The two candidates with the most first place votes are A and B.

50

Top Two Run-Off

	18	12	10	9	4	2
	A	B				
			B		B	
				B		B
	B	A	A	A	A	A

N=55, 28 needed to win.

Run-Off: A vs. B.

A: 18 votes.
B: 37 votes.

Top Two Run-Off

	18	12	10	9	4	2
	A	B				
			B		B	
				B		B
	B	A	A	A	A	A

N=55, 28 needed to win.

Run-Off: A vs. B.

A: 18 votes.
B: 37 votes.

B wins.

Comedy Central

The Tea Party in Arizona is holding an endorsement convention. The nominees are Carly Fiorina, Ted Cruz, and Kevin McCarthy. There are 100 delegates who cast rank ordered ballots.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Fiorina (37) and McCarthy (32) are the top two.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina			McCarthy
McCarthy	McCarthy	Fiorina McCarthy	McCarthy Fiorina	Fiorina

Fiorina (37) and McCarthy (32) are the top two.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina			McCarthy
McCarthy	McCarthy	Fiorina	McCarthy	
		McCarthy	Fiorina	Fiorina

Fiorina (37) and McCarthy (32) are the top two.

Fiorina wins 53 – 47.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Now suppose that in the morning before the election, Carly Fiorina met with some of the delegates and gave an impressive performance to the McCarthy block and that two of the McCarthy supporters decided to switch to Fiorina.

Comedy Central

27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Now suppose that in the morning before the election, Carly Fiorina met with some of the delegates and gave an impressive performance to the McCarthy block and that two of the McCarthy supporters decided to switch to Fiorina.

Comedy Central

27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Now the run-off is between Fiorina and Cruz (39 – 31 – 30)!

61

Comedy Central

27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	
Cruz	Cruz	Fiorina	Fiorina	Cruz Fiorina

Now the run-off is between Fiorina and Cruz (39 – 31 – 30)!

In the Fiorina vs. Cruz run-off, Cruz wins 61 – 39.

62

Comedy Central

27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	
Cruz	Cruz	Fiorina	Fiorina	Cruz Fiorina

Now the run-off is between Fiorina and Cruz (39 – 31 – 30)!

In the Fiorina vs. Cruz run-off, Cruz wins 61 – 39.

Increased support for Fiorina transforms her from a winner into a loser!

Decision Mechanisms

- Plurality

- Top two run-off

- Sequential elimination run-off

Sequential Elimination Run-Off

A top two run-off is a simplified elimination method. Along with plurality it is the most common method used in US political elections.

Some localities are replacing the top two run-off method with a ranked order system. For example, rank your top three choices.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Rank your top three choices

	Rank		
Option	1	2	3
A	○	○	○
B	○	○	○
C	○	○	○
D	○	○	○
E	○	○	○

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Rank your top three choices

	Rank		
Option	1	2	3
A	○	○	○
B	○	●	○
C	○	○	○
D	○	○	●
E	●	○	○

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D		
Second	D		B	C	B	C
Third		D			D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D
Third						

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C		B	C
Second			B	C		

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second			B			

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second			B			

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.
Still no majority. Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A		C	C		C
Second						

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.
Still no majority. Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A		C	C		C

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.
Still no majority. Eliminate B.

C wins 21-18.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the candidates

Option	Rank				
	1	2	3	4	5
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○
E	○	○	○	○	○

We now consider a full-ranked sequential elimination run-off.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the candidates

Option	Rank				
	1	2	3	4	5
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○
E	○	○	○	○	○

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the candidates

	Rank				
Option	1	2	3	4	5
A	○	○	○	○	●
B	○	●	○	○	○
C	○	○	○	●	○
D	○	○	●	○	○
E	●	○	○	○	○

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D		
Second	D		B	C	B	C
Third		D			D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D
Third	C	C	D	B	C	B
Fourth	B	A	A	A	A	A
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D
Third	C	C	D	B	C	B
Fourth	B	A	A	A	A	A
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C		B	C
Second			B	C		
Third	C	C		B	C	B
Fourth	B	A	A	A	A	A
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second	C	C	B	B	C	B
Third	B	A	A	A	A	A
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second	C	C	B	B	C	B
Third	B	A	A	A	A	A
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A		C	C		C
Second	C	C			C	
Third		A	A	A	A	A
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	C	C	C	C	C
Second	C	A	A	A	A	A
Third						
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

92

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	C	C	C	C	C
Second	C	A	A	A	A	A
Third						
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

C wins: 37-18 !

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First		B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B					

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	B	C	D	E	E
Second	E	E	B	C	B	C
Third	C	D	E	E	D	D
Fourth	B	C	D	B	C	B

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	B	C	D	E	E
Second	E	E	B	C	B	C
Third	C	D	E	E	D	D
Fourth	B	C	D	B	C	B

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.
Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	E	C	D	E	E
Second	E	D	E	C	D	C
Third	C	C	D	E	C	D

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.
Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	E	C	D	E	E
Second	E	D	E	C	D	C
Third	C	C	D	E	C	D

N=55, 28 needed to win.

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.
Eliminate B.
Eliminate C.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	E	E	D	E	E
Second	E	D	D	E	D	D

N=55, 28 needed to win.

D: 27 votes
E: 28 votes

E wins!

Coombs: Eliminate the "least desirable" candidate and then recount the votes.

Eliminate A.
Eliminate B.
Eliminate C.

Sequential Elimination Run-Off

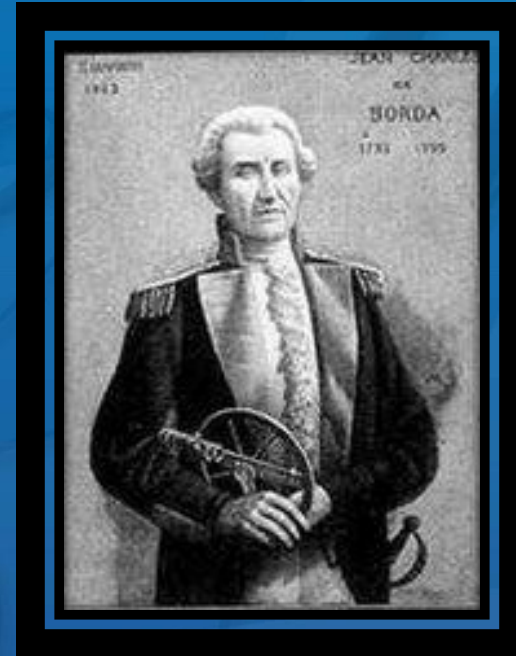The full sequential elimination method is used by the International Olympic Committee to decide the site of the Olympic Games.

In local political elections, "least fit" sequential elimination is more commonly known as Instant Runoff Voting (IRV).

Decision Mechanisms

- Plurality
- Top two run-off
- Sequential elimination run-off
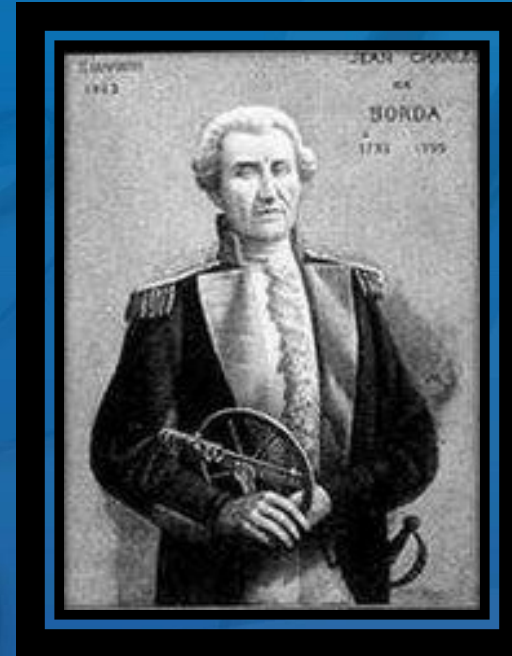- Borda Count

Borda Count

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Jean-Charles
Chevalier de Borda
1733 - 1799

Borda Count

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

First: 5 points.
Second: 4 points.
Third: 3 points.
Fourth: 2 points.
Fifth: 1 point.

Jean-Charles
Chevalier de Borda
1733 - 1799

Borda Count

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A: 127
B: 156
C: 158
D: 191
E: 146

Borda Count

4-3-2-1-0

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A: 127   A: 72
B: 156   B: 101
C: 158   C: 117
D: 191   D: 136
E: 146   E: 92

Borda Count

4-3-2-1-0

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A: 127   A: 72
B: 156   B: 101
C: 158   C: 117
D: 191   D: 136
E: 146   E: 92

D wins!

Borda Count

The Borda Count is used in these situations:

- Heisman Trophy
- AL and NL MVP awards
- Country Music Vocalist of the year.

Borda Count

Problem: The Borda Count can violate the Majority Criterion of Fairness.

Borda Count

Problem: The Borda Count can violate the Majority Criterion of Fairness.

	6	2	3
First	A	B	C
Second	B	C	D
Third	C	D	B
Fourth	D	A	A

Borda Count

Problem: The Borda Count can violate the Majority Criterion of Fairness.

	6	2	3
First	A	B	C
Second	B	C	D
Third	C	D	B
Fourth	D	A	A

Borda Points:

A: 29
B: 32
C: 30
D: 19

B wins!

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

115

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

The students ask, Why vanilla? Were they out of chocolate?

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

The students ask, Why vanilla? Were they out of chocolate?

The teacher says, Well, no. They had chocolate. I got vanilla because they were out of strawberry.

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

The students ask, Why vanilla? Were they out of chocolate?

The teacher says, Well, no. They had chocolate. I got vanilla because they were out of strawberry.

How could this happen?

Comedy Central

4	2	5
Strawberry Vanilla Chocolate	Vanilla Chocolate Strawberry	Chocolate Strawberry Vanilla

Comedy Central

4	2	5
Strawberry Vanilla Chocolate	Vanilla Chocolate Strawberry	Chocolate Strawberry Vanilla

Plurality, top two run-off, either form of elimination, and the Borda Count all produce Chocolate as the winner.

Comedy Central

4	2	5
Strawberry Vanilla Chocolate	Vanilla Chocolate Strawberry	Chocolate Strawberry Vanilla

But since they were out of strawberry, then the teacher eliminated strawberry from the count.

Comedy Central

4	2	5
Vanilla Chocolate	Vanilla Chocolate	Chocolate Vanilla

But since they were out of strawberry, then the teacher eliminated strawberry from the count.

So, Vanilla wins 6 – 5!

Decision Mechanisms

- Plurality
- Top two run-off
- Sequential elimination run-off
- Borda Count
- Condorcet

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.

Marie Jean Antoine Nicholas de Caritat, Marquis de Condorcet 1743 - 1794

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.

Marie Jean Antoine Nicholas de Caritat, Marquis de Condorcet
1743 - 1794

Note: A is a Condorcet Loser!

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.

A vs. E   B vs. E   C vs. E   D vs. E
18   37   22   33   19   36   27   28

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.

A vs. E   B vs. E   C vs. E   D vs. E

18   37   22   33   19   36   27   28   E wins!

The Winner

- Plurality
- Top two run-off
- IRV Sequential elimination
- Borda Count
- Condorcet

The Winner

- Plurality: A

- Top two run-off: B

- IRV Sequential elimination: C

- Borda Count: D

- Condorcet: E

The Winner

- Plurality: A
- Top two run-off: B
- IRV Sequential elimination: C; Coombs: E
- Borda Count: D
- Condorcet: E

Note: "least desirable" sequential elimination (Coombs Method) for a fully ranked ballot will always select the Condorcet winner if there is one.

The Question

Is there a fair way to decide?

The Question

Is there a fair way to decide?

What do you mean by fair?

Fairness Axioms

Fairness Axioms

- Individual Sovereignty (free ballot)

Fairness Axioms

- Individual Sovereignty (free ballot)
- Majority Rule

Fairness Axioms

- Individual Sovereignty (free ballot)
- Majority Rule
- Condorcet Rule

Fairness Axioms

- Individual Sovereignty (free ballot)
- Majority Rule
- Condorcet Rule
- An increase in support does not damage one's chance of winning.

Fairness Axioms

- Individual Sovereignty (free ballot)
- Majority Rule
- Condorcet Rule
- An increase in support does not damage one's chance of winning.
- If a losing candidate drops out and voter preferences are constant, then the election result is not changed.

Arrow's Theorem

There is no voting system that can satisfy all requirements in all cases.
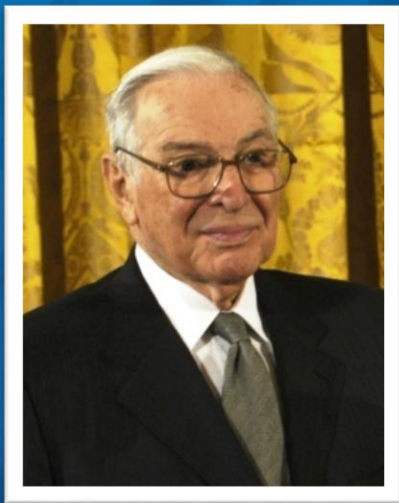
Kenneth Arrow
Nobel Prize in Economics 1972

Condorcet Paradox

A simple way to illustrate the difficulty is to consider the following example.

Voter 1: A, B, C
Voter 2: B, C, A
Voter 3: C, A, B

Condorcet Paradox

A simple way to illustrate the difficulty is to consider the following example.

Voter 1: A, B, C
Voter 2: B, C, A
Voter 3: C, A, B

In pairwise voting,

A beats B, 2 to 1;
B beats C, 2 to 1;
yet,   C beats A, 2 to 1.

Pairwise voting may not be transitive.

Points to Ponder

Even though "perfection" isn't possible, improvement may be possible.

Approval Voting

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Assume that each voter would approve of their top two choices.

Approval Voting

	A	B	C	D	E
First	18	12	10	9	6
Second	0	14	11	18	12
Total	18	26	21	27	18

Still no one gets a majority (N = 55; 28 needed to win).
The top-two runoff would run off B (26 votes) vs. D (29 votes).
D wins!

144

Approval Voting

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Assume that each voter would approve of their top three choices.

145

Approval Voting

	A	B	C	D	E
First	18	12	10	9	6
Second	0	14	11	18	12
Third	0	0	0	18	37
Total	18	26	21	45	55

Only D and E get a majority; E is "unanimously approved".
E wins!

The Ballot

Structure

- **List Only**
  - ➢ Vote for One
  - ➢ Vote for Approved

- **List and Rank**
  - ➢ Partial
  - ➢ Full

Decision

- **List Only**
  - ➢ Plurality
  - ➢ Run-Off

- **List and Rank**
  - ➢ IRV
  - ➢ Coombs
  - ➢ Borda
  - ➢ Condorcet
  - ➢ Approval

Case Study

The Australian Electoral System

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets ½; son #2 gets 1/3; son #3 gets 1/9.

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets ½; son #2 gets 1/3; son #3 gets 1/9.

The father dies. The sons are in a quandary.

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets ½; son #2 gets 1/3; son #3 gets 1/9.

The father dies. The sons are in a quandary.

½ x 17 = 8 ½
$^1/_3$ x 17 = 5 $^2/_3$
$^1/_9$ x 17 = 1 $^8/_9$

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets ½; son #2 gets 1/3; son #3 gets 1/9.

The father dies. The sons are in a quandary.

$$\tfrac{1}{2} \times 17 = 8\ \tfrac{1}{2}$$
$$\tfrac{1}{3} \times 17 = 5\ \tfrac{2}{3}$$
$$\tfrac{1}{9} \times 17 = 1\ \tfrac{8}{9}$$

A wise man notes the dilemma of the sons and contributes his camel, yielding 18 camels to be distributed.

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets ½; son #2 gets 1/3; son #3 gets 1/9.

The father dies. The sons are in a quandary.

$\frac{1}{2} \times 17 = 8\frac{1}{2}$ $\frac{1}{2} \times 18 = 9$

$\frac{1}{3} \times 17 = 5\frac{2}{3}$ $\frac{1}{3} \times 18 = 6$

$\frac{1}{9} \times 17 = 1\frac{8}{9}$ $\frac{1}{9} \times 18 = 2$

Each son gets more than his inheritance.

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets ½; son #2 gets 1/3; son #3 gets 1/9.

The father dies. The sons are in a quandary.

$$½ \times 17 = 8 ½ \qquad\qquad ½ \times 18 = 9$$
$$⅓ \times 17 = 5 ⅔ \qquad\qquad ⅓ \times 18 = 6$$
$$⅑ \times 17 = 1 ⅛\!/_9 \qquad\qquad ⅑ \times 18 = 2$$

Each son gets more than his inheritance.

And the wise man rides off on the left over camel.

Thank You

Problems

In case of a tie in a political election:

http://fivethirtyeight.com/datalab/the-2014-elections-that-ended-in-a-tie/

http://www.theatlantic.com/politics/archive/2012/11/when-a-state-election-can-be-literally-determined-by-a-coin-toss/265413/

Problems

Problems involving *We The People* include:

Problems

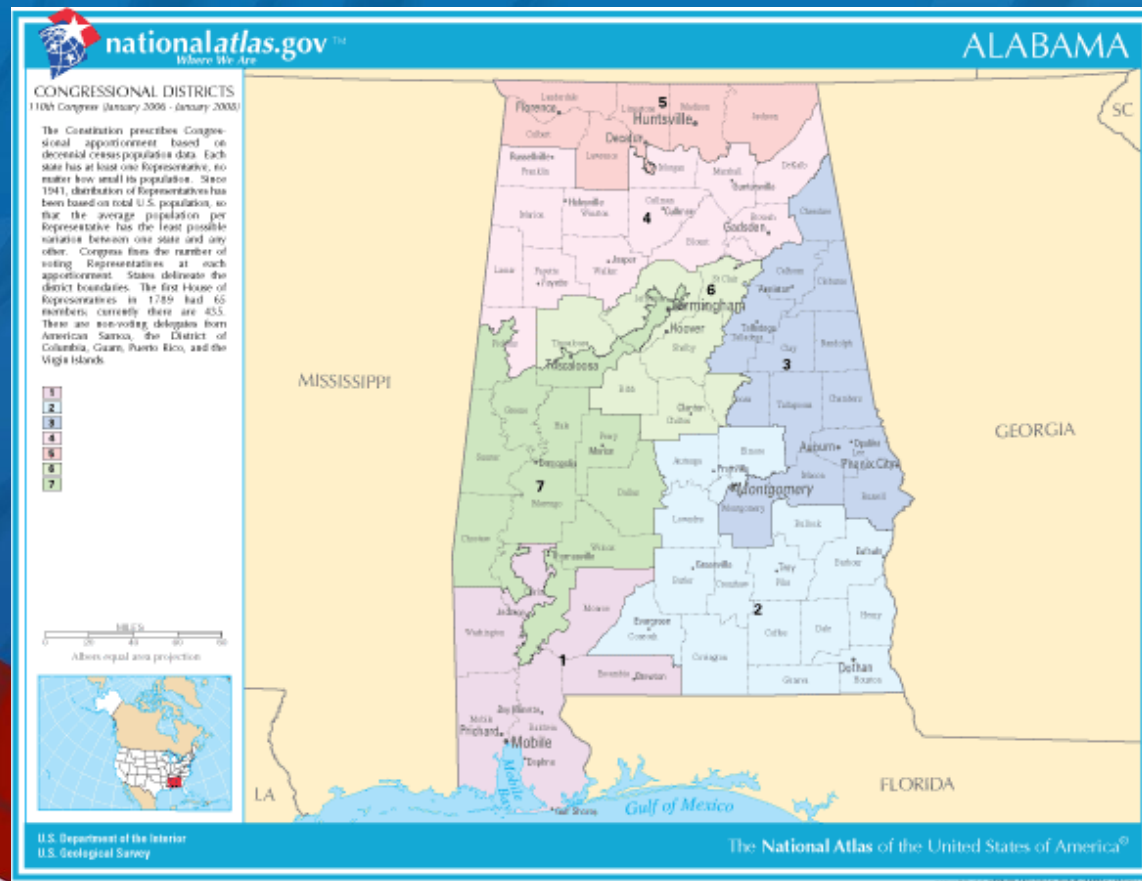Problems involving *We The People* include:

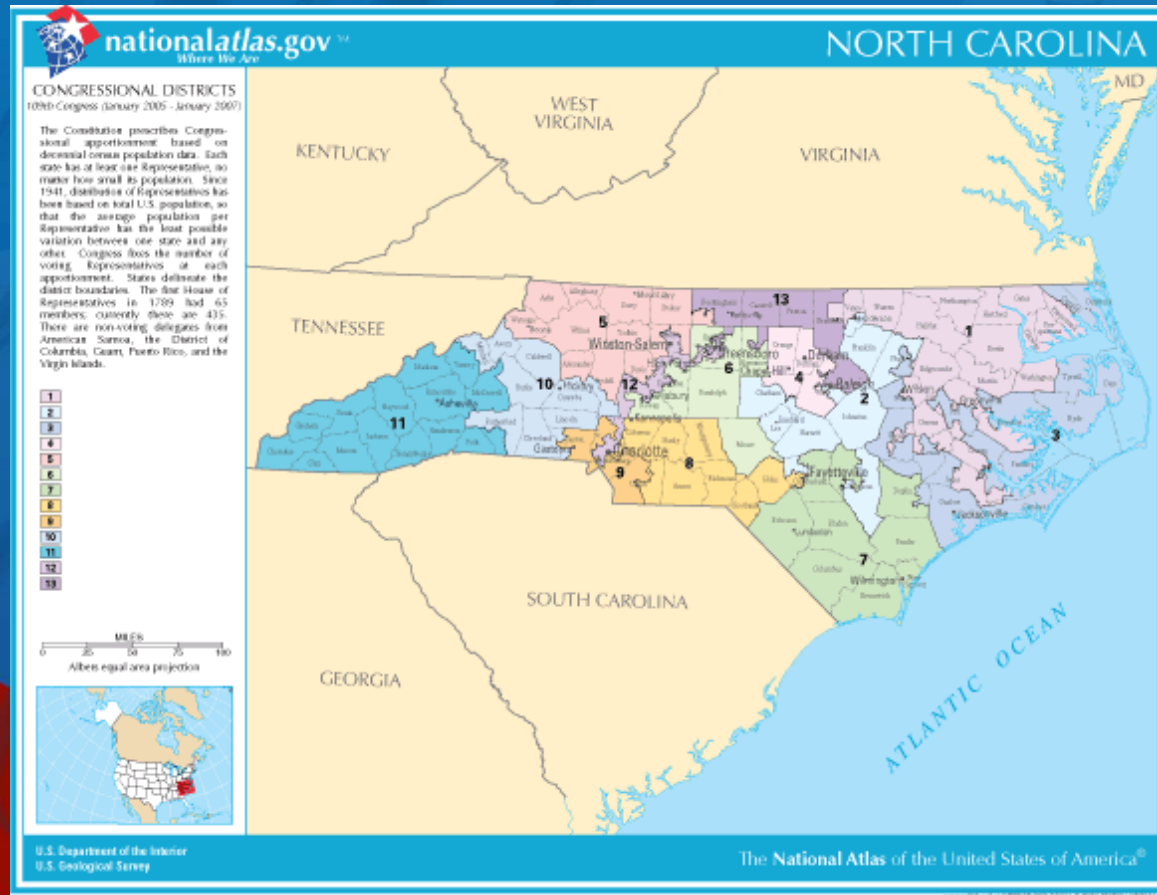➢ Census: who is "enumerated."

Problems

Problems involving *We The People* include:

➢ Census: who is "enumerated."
➢ Districting.

Gerrymandering

Gerrymandering

Problems

Problems involving *We The People* include:

➢ Census: who is "enumerated."
➢ Districting.
➢ Suffrage: who is allowed to vote.

Problems

Problems involving *We The People* include:

➤ Census: who is "enumerated."
➤ Districting.
➤ Suffrage: who is allowed to vote.
➤ Voting: the mechanism of voting.

Problems

Problems involving *We The People* include:

➢ Census: who is "enumerated."
➢ Districting.
➢ Suffrage: who is allowed to vote.
➢ Voting: the mechanism of voting.
➢ Decision: how does one decide the winner?