

Congressional Apportionment

Bringing Down the House

Charles Biles, Ph.D.

Mathematics 103I: Contemporary Mathematics

Humboldt State University

Spring Semester 2017

website: nia977.wix.com/drbcap

“... no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, ...”

James Madison
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The Apportionment Problem

Determine how many seats in the U.S.
House of Representatives each state gets.

CONGRESSIONAL SEATS

2010
OFFICIAL RESULTS



GAINED
LOST
NO CHANGE

United States
Census
2010

(US apportionment population = 309,183,463)/435 ≈ 710,767

<http://www.census.gov/2010census/data/apportionment-data.php>

Apportionment History

Last time we looked at the first two periods of congressional apportionment history.

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The Basic Divisor Method: 1790-1840.

The Quota Method: 1850-1900.

1910

Apportionment based on the 1910 census came from another mutation in apportionment methodology.

Congress abandoned the Quota Method and used a modified divisor method.

Modified Divisor Methods

Step 1. Select the House size h .

Step 2. Select a constituency, d .

Smart start:

$$d = (\text{national population})/h$$

Step 3. Calculate $q = p/d$ and $n = \text{int}(q)$.

Step 4. Let $a = \max(1, \text{rnd}(q)) \in \{n, n+1\}$

Step 5. If apportionments add to h , then DONE;
else, modify d and GO TO Step 3.

Modified Divisor Methods

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1830: Jefferson: $\text{rnd}(q) = n$

Adams: $\text{rnd}(q) = n + 1$

Dean: $\text{rnd}(q) = n + 1$ iff $q \geq \text{HM}(n, n+1)$

Webster: $\text{rnd}(q) = n + 1$ iff $q \geq \text{AM}(n, n+1)$

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1910 : $h = 433$ and Webster's method of rounding.

1920 Census

In the 1920 decade there was so much confusion and politics that for the only time in U. S. History no census-based re-apportionment act was passed.

Congress could not agree on either the size of the House or on the method of apportionment. Further, the politics of prohibition played a significant role: the dries would not consider any allocation giving the wets more power.

Today

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Dean:

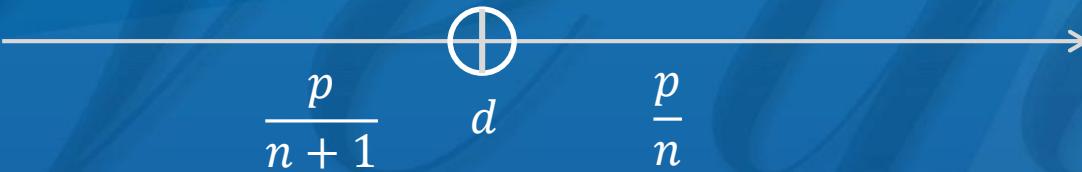
$$\frac{p}{n+1} \quad d \quad \frac{p}{n}$$


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H-H:

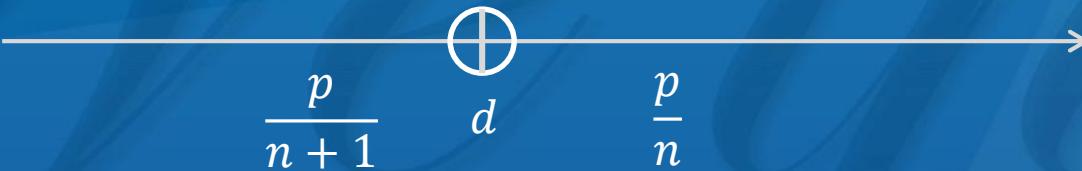


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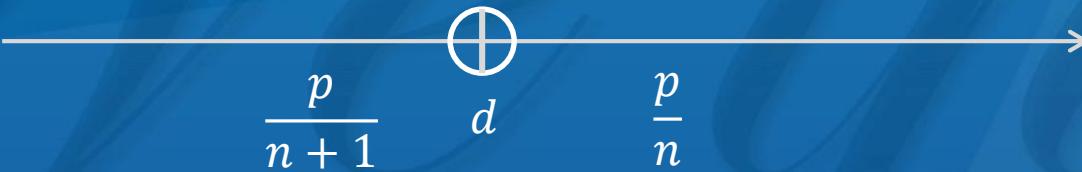
Criterion: $a = n+1$ iff $\frac{d}{\left(\frac{p}{n+1}\right)} \leq \frac{\left(\frac{p}{n}\right)}{d}$

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There are no perfect apportionment methods.

Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

They Mean Well

A modified ***divisor*** method first fixes the House size, then seeks a divisor that when the state's quotients are rounded and summed, the house size is achieved.

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Webster: arithmetic mean $\frac{n + (n + 1)}{2} = n + \frac{1}{2}$

Huntington-Hill: geometric mean $\sqrt{n(n + 1)}$

Dean: harmonic mean $\frac{2}{\frac{1}{n} + \frac{1}{n + 1}} = \frac{2n(n + 1)}{2n + 1}$

Montana

In the 1990 apportionment, Montana lost one of its two seats it held for 80 years. In 1991 MT filed suit in federal district court (MT vs. US Dept Commerce).

MT argued the H-H method is unconstitutional and that either Dean's or Adams's method should be used. The federal judges voted 2-1 in favor of MT.

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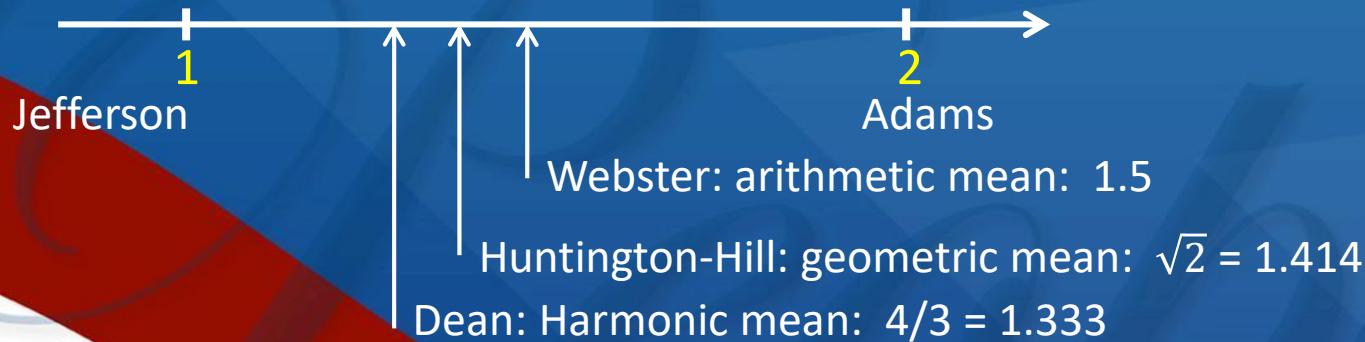
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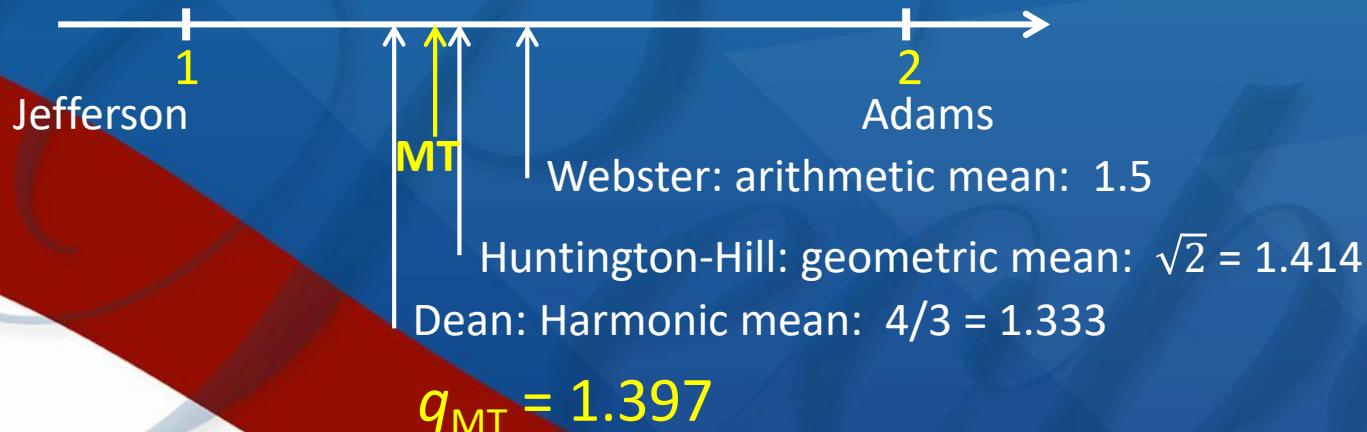
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Today

https://www.census.gov/library/video/census_apportionment_machine.html

Today

Today the Census Bureau obtains apportionments using a priority technique of calculation rather than an ad-hoc technique of calculation.

2010 Census

<http://www.census.gov/data/tables/2010/dec/2010-apportionment-data.html>

An Average Lesson

1. How to average two different positive numbers.
2. How to round a positive decimal number.

An Average Lesson

1. The average of a and b where $0 < a < b$.

$$\text{ave}(a,b) = \max(a,b) = b$$

$$\min(a,b) = a$$

$$\text{AM}(a,b) = (a + b)/2$$

$$\text{GM}(a,b) = \sqrt{a \times b}$$

$$\text{HM}(a,b) = \frac{2}{\left(\frac{1}{a} + \frac{1}{b}\right)} = \frac{2ab}{a + b}$$

Priority Numbers

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$$PN(n) = \frac{\text{population}}{\text{ave}(n,n+1)}$$

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Dean: $\text{HM}(n,n+1)$ HM

Huntington-Hill: $\text{GM}(n,n+1)$ EP

Webster: $\text{AM}(n,n+1)$ MF

Adams: $\min(n,n+1)$ SD

Priority Numbers

Census 1790		
State	Population	Seats
Connecticut	236841	1
Delaware	55540	1
Georgia	70835	1
Kentucky	68705	1
Maryland	278514	1
Massachusetts	475327	1
New Hampshire	141822	1
New Jersey	179570	1
New York	331589	1
North Carolina	353523	1
Pennsylvania	432879	1
Rhode Island	68446	1
South Carolina	206236	1
Vermont	85533	1
Virginia	630560	1
US	3615920	15

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$$PN(1) = p/\sqrt{1 \times 2} = p/\sqrt{2}$$

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$$PN(2) = p/\sqrt{2 \times 3} = p/\sqrt{6}$$

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$$PN(1) = p/\sqrt{1 \times 2} = p/\sqrt{2}$$

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$$PN(3) = p/\sqrt{3 \times 4} = p/\sqrt{12}$$

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$$PN(3) = p/\sqrt{3 \times 4} = p/\sqrt{12}$$

$$PN(4) = p/\sqrt{4 \times 5} = p/\sqrt{20}$$

Priority Numbers

Census 1790		Seats	Priority Numbers H-H			
State	Population		sqrt(2)	sqrt(6)	sqrt(12)	sqrt(20)
Connecticut	236841	1	167471	96689	68370	52959
Delaware	55540	1	39272	22674	16033	12419
Georgia	70835	1	50087	28918	20448	15839
Kentucky	68705	1	48581	28048	19833	15362
Maryland	278514	1	196939	113702	80400	62277
Massachusetts	475327	1	336106	194051	137215	106286
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Pennsylvania	432879	1	306091	176722	124961	96794
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State	Population		sqrt(2)	sqrt(6)	sqrt(12)	sqrt(20)
Connecticut	236841	1	167471	96689	68370	52959
Delaware	55540	1	39272	22674	16033	12419
Georgia	70835	1	50087	28918	20448	15839
Kentucky	68705	1	48581	28048	19833	15362
Maryland	278514	1	196939	113702	80400	62277
Massachusetts	475327	2		194051	137215	106286
New Hampshire	141822	1	100283	57898	40940	31712
New Jersey	179570	1	126975	73309	51837	40153
New York	331589	1	234468	135370	95721	74145
North Carolina	353523	1	249978	144325	102053	79050
Pennsylvania	432879	2		176722	124961	96794
Rhode Island	68446	1	48398	27942	19758	15304
South Carolina	206236	1	145830	84195	59535	46115
Vermont	85533	1	60480	34918	24691	19125
Virginia	630560	3		257425	182026	140997
US	3615920	19				

Priority Numbers

16 VA 2
 17 MA 2
 18 PA 2
 19 VA 3

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	Vermont	85533	1	60480	34918	24691	19125
	Virginia	630560	4				140997
	US	3615920	24				

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Who's next???

Today

$$A_n = \frac{P}{ave(n, n + 1)}$$

$$A_n = \frac{P}{\sqrt{n \times (n + 1)}}$$

The Future: Reform?

Four Proposals:

The Future: Reform?

Four Proposals:

- Thirty-thousand.org
- The Wyoming Rule
- Neubauer and Gartner
- Current method with rounding by Webster's Method.

thirty-thousand.org

Here's an example of a concerned group:

<http://www.thirty-thousand.org/>

thirty-thousand.org

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Comment: This leads to a House with 10283 representatives.

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Comment: This leads to a House with 10283 representatives.

CA: 1244 seats!

thirty-thousand.org

Here's an example of a concerned group:

<http://www.thirty-thousand.org/>

Thirty-thousand.org advocates 50000/representative.
This leads to a House with 6181 representatives using
Webster's method of rounding.
California gets 747 seats.

The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).

http://en.wikipedia.org/wiki/Wyoming_Rule

<http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/>

The Wyoming Rule

Here are the results of applying the WY Rule
to the 2000 and 2010 censuses.

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2000 smallest state: WY, 493782.

$h = 569$ Huntington-Hill

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2000 smallest state: WY, 493782.

$h = 569$ Huntington-Hill

2010 smallest state: WY, 563626

$h = 543$ Dean HI

$h = 542$ Huntington-Hill

$h = 540$ Webster NJ, SD

A Proposal

A Proposal for Apportioning the House

Michael G. Neubauer, CSU Northridge, Mathematics

Margo G. (Gartner) Carr, Cerro Coso Community College

...the problem of finding a “good” house size and “right” apportionment method are best considered together.

Source: PSC 44(1), January 2011: 1—3.

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...the problem of finding a “good” house size and “right” apportionment method are best considered together.

Definition. A House size is *agreeable* means that the apportionments by the methods of Hamilton, Dean, Huntington-Hill, and Webster all agree.

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Proposal. From the 2000 census, $h = 435$ was not agreeable. The first agreeable House size greater than 435 is $h = 477$.

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Source: PSC 44(1), January 2011: 1—3.

Webster's Method

The simplest reform would be to replace the geometric mean of decimal rounding in the Huntington-Hill method by the arithmetic mean of decimal rounding in Webster's method.

Webster's Method

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The research of Balinski and Young has produced the following two key results. Since the Alabama paradox is a deal-breaker, then congressional apportionment must be based on a divisor method. Further, Webster's is the only rounding method that is unbiased towards either larger or smaller states.

Thank You

It is time that I took my seat in this House!

<http://www.nia977.wix.com/drbcap>

Bonus Resources

Related Problems

Other problems related to apportionment include:

One Voter, One Vote: The Apportionment of Congressional Seats Reconsidered

Author(s): Howard A. Scarrow

Source: Polity, Vol. 22, No. 2 (Winter, 1989), pp. 253-268

Published by: Palgrave Macmillan Journals

Stable URL: <http://www.jstor.org/stable/3234834> .

Related Problems

Other problems related to apportionment include:

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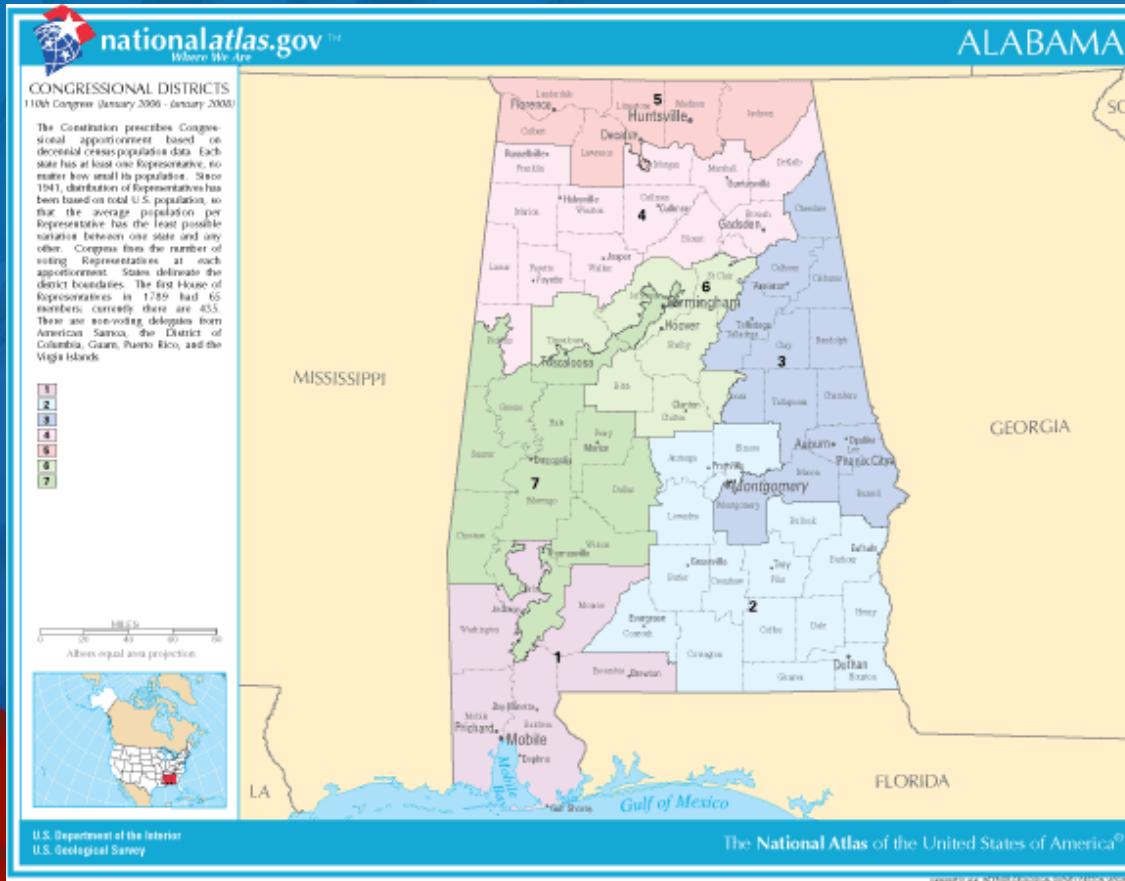
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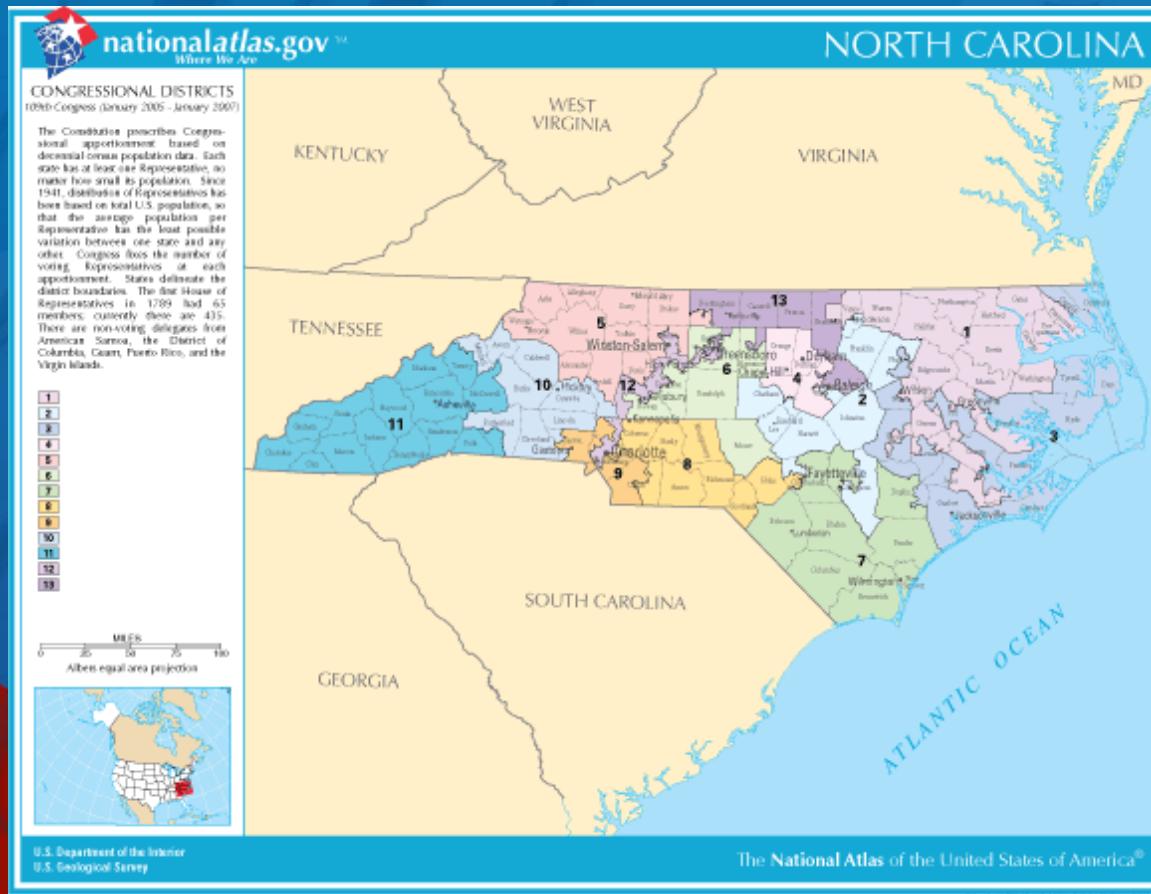
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Gerrymandering



<http://www.nationalatlas.gov/printable/congress.html#al>

Gerrymandering



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- Suffrage: who is allowed to vote.

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- Census: who is “enumerated.”
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- Suffrage: who is allowed to vote.
- Voting: the mechanism of voting.

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US History

For any serious research of U.S. history, one must know about the Journals of Congress which includes the House Journal and the Senate Journal:

<http://memory.loc.gov/ammem/amlaw/lwhj.html>

Apportionment Problems

On appeal, the U.S. Supreme Court ruled unanimously that the H-H method was constitutional. The district court's decision was overturned.

<http://www.law.cornell.edu/supct/html/91-860.ZS.html>

<http://caselaw.lp.findlaw.com/scripts/getcase.pl?court=US&vol=503&invol=442>

Washington's Veto

United States [Philadelphia] April 5 1792.

Gentlemen of the House of Representatives

I have maturely considered the Act passed by the two Houses, intitled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.

First—The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.

Second—The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the separate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.

George Washington.

Copy, DNA: RG 233, Second Congress, 1791–1793, Records of Legislative Proceedings, Journals; LB, DLC:GW. (from Philander Chase, et al., eds., *The Papers of George Washington, Presidential Series, Vol. 10: March–August 1792* [Charlottesville, Va., 2002], 213–14).

First Apportionment Act

CHAP. XXIII.—*An Act for apportioning Representatives among the several States, according to the first enumeration.*

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachusetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

APPROVED, April 14, 1792.

1790: Why 33000?

State	Population	<i>d</i> = 30000	31000	32000	33000	34000	35000	36000	37000	38000	39000	40000
CN	236841	0.8947	0.6400	0.4013	0.1770	0.9659	0.7669	0.5789	0.4011	0.2327	0.0728	0.9210
DE	55540	0.8513	0.7916	0.7356	0.6830	0.6335	0.5869	0.5428	0.5011	0.4616	0.4241	0.3885
GA	70835	0.3612	0.2850	0.2136	0.1465	0.0834	0.0239	0.9676	0.9145	0.8641	0.8163	0.7709
KY	68705	0.2902	0.2163	0.1470	0.0820	0.0207	0.9630	0.9085	0.8569	0.8080	0.7617	0.7176
MD	278514	0.2838	0.9843	0.7036	0.4398	0.1916	0.9575	0.7365	0.5274	0.3293	0.1414	0.9629
MA	475327	0.8442	0.3331	0.8540	0.4038	0.9802	0.5808	0.2035	0.8467	0.5086	0.1879	0.8832
NH	141822	0.7274	0.5749	0.4319	0.2976	0.1712	0.0521	0.9395	0.8330	0.7322	0.6365	0.5456
NJ	179570	0.9857	0.7926	0.6116	0.4415	0.2815	0.1306	0.9881	0.8532	0.7255	0.6044	0.4893
NY	331589	0.0530	0.6964	0.3622	0.0482	0.7526	0.4740	0.2108	0.9619	0.7260	0.5023	0.2897
NC	353523	0.7841	0.4040	0.0476	0.7128	0.3977	0.1007	0.8201	0.5547	0.3032	0.0647	0.8381
PA	432879	0.4293	0.9638	0.5275	0.1175	0.7317	0.3680	0.0244	0.6994	0.3916	0.0995	0.8220
RI	68446	0.2815	0.2079	0.1389	0.0741	0.0131	0.9556	0.9013	0.8499	0.8012	0.7550	0.7112
SC	206236	0.8745	0.6528	0.4449	0.2496	0.0658	0.8925	0.7288	0.5739	0.4273	0.2881	0.1559
VT	85533	0.8511	0.7591	0.6729	0.5919	0.5157	0.4438	0.3759	0.3117	0.2509	0.1932	0.1383
VA	630560	0.0187	0.3406	0.7050	0.1079	0.5459	0.0160	0.5156	0.0422	0.5937	0.1682	0.7640
US	3615920	8.5307	8.6426	6.9975	4.5733	6.3506	7.3120	9.4422	9.7276	8.1558	5.7159	9.3980

Unrepresented: 255920 267920 223920 150920 215920 255920 339920 359920 309920 222920 375920

Alabama Paradox

How is this possible?

State	House 299	House 300
AL	7.646	7.671
TX	9.640	9.672
IL	18.640	18.702

With the House size at 299, Alabama was the last state to be allotted an extra representative to make the House size because of its decimal. When the House size was increased to 300, all states' quotas were increased by 0.33%. And there were two states that got the extra representatives; and, this time, Texas and Illinois beat out Alabama.

US Census Bureau

The U.S. Census Bureau is housed within the Department of Commerce.

Check out the U.S. Census Bureau for what it says about apportionment.

<http://www.census.gov/>

Summary 7-page brochure:

<http://www.census.gov/prod/cen2010/briefs/c2010br-08.pdf>

History of Legislation:

http://www.census.gov/history/www/reference/apportionment/apportionment_legislation_1790 - 1830.html

More!

For playing around, learning or teaching:

<http://www.cut-the-knot.org/ctk/Democracy.shtml>