## The Apportionment Problem Bringing Down the House

Charles Biles, Ph.D.
United States Government
Academy of the Redwoods
10 March 2017
website: nia977.wix.com/drbcap
"... no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, ..."

James Madison
The Federalist 55

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## The Apportionment Problem

Determine how many seats in the U.S. House of Representatives each state gets.


## CONGRESSIONAL SEATS


(US apportionment population $=309,183,463$ )/435 $\approx 710,767$


## Today

## https://www.census.gov/library/video/census appor tionment machine.html

## Today

Today the Census Bureau obtains apportionments using a priority technique of calculation rather than an ad-hoc technique of calculation.

## An Average Lesson

1. How to average two different positive numbers.
2. How to round a positive decimal number.

## An Average Lesson

1. The average of $a$ and $b$ where $0<a<b$.

$$
\begin{aligned}
\operatorname{ave}(a, b)=\max (a, b) & =b \\
\min (a, b) & =a \\
\operatorname{AM}(a, b) & =(a+b) / 2 \\
\operatorname{GM}(a, b) & =\sqrt{a \times b} \\
\operatorname{HM}(a, b) & =\frac{2}{\left(\frac{1}{a}+\frac{1}{b}\right)}=\frac{2 a b}{a+b}
\end{aligned}
$$

# Ad-hoc Modified Divisor 

Step 1. Decide the House size: $h$.
Step 2. Apply a basic divisor method to obtain the preset $h$.

## Serial Distribution

Step 1. Award 1 seat to each state. Today this distributes 50 seats.

Step 2. Then award the $51^{\text {st }}$ seat, $52^{\text {nd }}$ seat, $53^{\text {rd }}$ seat, etc., according to a list of priority numbers.

## Priority Numbers

| 2010 Census |  |  |  |
| :--- | :--- | :--- | :--- |
| Seat | Priority | State | Apportionment |
| 434 | 711308 | CA | 53 |
| 435 | 710231 | MN | 8 |
| 436 | 709063 | NC | 14 |
| 437 | 708459 | MO | 9 |
| 438 | 706337 | NY | 28 |
| 439 | 70564 | NJ | 13 |
| 440 | 703158 | MT | 2 |

## Priority Numbers

$$
A_{n} \longrightarrow \operatorname{PN}(n)=\frac{\text { population }}{\operatorname{ave}(n, n+1)}
$$

## Priority Numbers


where ave $(n, n+1)=$
Jefferson: $\max (n, n+1)$
Dean: HM $(n, n+1)$
Huntington-Hill: GM( $n, n+1$ )
Webster: AM $(n, n+1)$
Adams: $\min (n, n+1)$

## Priority Numbers


where ave $(n, n+1)=$
Jefferson: $\max (n, n+1)$ Largest Divisors
Dean: HM $(n, n+1)$ Harmonic Means
Huntington-Hill: GM $(n, n+1)$ Equal Proportions
Webster: AM $(n, n+1)$ Major Fractions
Adams: min $(n, n+1)$ Smallest Divisors

## Today

$$
\begin{aligned}
& A_{n}=\frac{P}{\operatorname{ave}(n, n+1)} \\
& A_{n}=\frac{P}{\sqrt{n \times(n+1)}}
\end{aligned}
$$

## Priority Numbers

| Census 1790 |  |
| ---: | ---: |
| State | Population |
| Connecticut | 236841 |
| Delaware | 55540 |
| Georgia | 70835 |
| Kentucky | 68705 |
| Maryland | 278514 |
| Massassachutts | 475327 |
| New Hampshire | 141822 |
| New Jersey | 179570 |
| New York | 331589 |
| North Carolina | 353523 |
| Pennsylvania | 432879 |
| Rhode Island | 68446 |
| South Carolina | 206236 |
| Vermont | 85533 |
| Virginia | 630560 |
| United States | 3615920 |

## Priority Numbers

| Census 1790 |  |  |
| ---: | ---: | ---: |
| State | Population | Seats |
| Connecticut | 236841 | 1 |
| Delaware | 55540 | 1 |
| Georgia | 70835 | 1 |
| Kentucky | 68705 | 1 |
| Maryland | 278514 | 1 |
| Massassachutts | 475327 | 1 |
| New Hampshire | 141822 | 1 |
| New Jersey | 179570 | 1 |
| New York | 331589 | 1 |
| North Carolina | 353523 | 1 |
| Pennsylvania | 432879 | 1 |
| Rhode Island | 68446 | 1 |
| South Carolina | 206236 | 1 |
| Vermont | 85533 | 1 |
| Virginia | 630560 | 1 |
| United States | 3615920 | 15 |

## Priority Numbers

| Census 1790 |  |  |
| ---: | ---: | ---: |
| State | Population | Seats |
| Connecticut | 236841 | 1 |
| Delaware | 55540 | 1 |
| Georgia | 70835 | 1 |
| Kentucky | 68705 | 1 |
| Maryland | 278514 | 1 |
| Massassachutts | 475327 | 1 |
| New Hampshire | 141822 | 1 |
| New Jersey | 179570 | 1 |
| New York | 331589 | 1 |
| North Carolina | 353523 | 1 |
| Pennsylvania | 432879 | 1 |
| Rhode Island | 68446 | 1 |
| South Carolina | 206236 | 1 |
| Vermont | 85533 | 1 |
| Virginia | 630560 | 1 |
| United States | 3615920 | 15 |

Huntington - Hill
$\mathrm{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$

## Priority Numbers

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 1 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 1 | 445873 |
| United States | 3615920 | 15 |  |

## Priority Numbers

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 1 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 1 | 445873 |
| United States | 3615920 | 15 |  |

## Priority Numbers

16 VA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 1 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 445873 |
| United States | 3615920 | 16 |  |

## Priority Numbers

16 VA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 1 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 |  |
| United States | 3615920 | 16 |  |

## Priority Numbers

16 VA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| PN $(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |  |  |  |
| Massassachutts | 475327 | 1 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 |  |
| United States | 3615920 | 16 |  |

## Priority Numbers

16 VA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| PN $(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |  |  |  |
| Massassachutts | 475327 | 1 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 16 |  |

## Priority Numbers

16 VA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
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| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 16 |  |

Huntington - Hill

$$
\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}
$$

$$
\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}
$$

## Priority Numbers

16 VA 2
17 MA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 2 | 336106 |
| New Hampshire | 141822 | 1 | 100283 |
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| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 17 |  |

Huntington - Hill

$$
\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}
$$

$$
\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}
$$

## Priority Numbers



| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 2 |  |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 1 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 17 |  |

Huntington - Hill

$$
\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}
$$

$\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}$

## Priority Numbers

|  | Census 1790 |  |  | H-H | Huntington - Hill |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | State | Population | Seats | Priority |  |
| 16 VA 2 | Connecticut | 236841 | 1 | 167471 |  |
| 17 MA 2 | Delaware | 55540 | 1 | 39272 |  |
|  | Georgia | 70835 | 1 | 50087 | $\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |
|  | Kentucky | 68705 | 1 | 48581 |  |
|  | Maryland | 278514 | 1 | 196939 | $\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}$ |
|  | Massassachutts | 475327 | 2 | 194051 |  |
|  | New Hampshire | 141822 | 1 | 100283 |  |
|  | New Jersey | 179570 | 1 | 126975 |  |
|  | New York | 331589 | 1 | 234468 |  |
|  | North Carolina | 353523 | 1 | 249978 |  |
|  | Pennsylvania | 432879 | 1 | 306091 |  |
|  | Rhode Island | 68446 | 1 | 48398 |  |
|  | South Carolina | 206236 | 1 | 145830 |  |
|  | Vermont | 85533 | 1 | 60480 |  |
|  | Virginia | 630560 | 2 | 257425 |  |
|  | United States | 3615920 | 17 |  |  |

## Priority Numbers

16 VA 2
17 MA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 2 | 194051 |
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| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
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| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
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| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 17 |  |

Huntington - Hill
$\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$
$\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}$

## Priority Numbers

$$
\begin{array}{lll}
16 & \text { VA } & 2 \\
17 & \text { MA } & 2 \\
18 & \text { PA } & 2
\end{array}
$$

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| PN |  |  |  |
| PN $(2)=p / \sqrt{1 \times 2}=p / \sqrt{2 \times 3}=p / \sqrt{6}$ |  |  |  |
| Massassachutts | 475327 | 2 | 194051 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 2 | 306091 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 18 |  |

## Priority Numbers

$$
\begin{array}{lll}
16 & \text { VA } & 2 \\
17 & \text { MA } & 2 \\
18 & \text { PA } & 2
\end{array}
$$



## Priority Numbers

16 VA 2
17 MA 2
18 PA 2

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| PN $(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |  |  |  |
| Massassachutts | 475327 | 2 | 194051 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 2 | 176722 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 18 |  |

## Priority Numbers

$$
\begin{array}{lll}
16 & \text { VA } & 2 \\
17 & \text { MA } & 2 \\
18 & \text { PA } & 2
\end{array}
$$

| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| PN |  |  |  |
| PN $(2)=p / \sqrt{1 \times 2}=p / \sqrt{2 \times 3}=p / \sqrt{6}$ |  |  |  |
| Massassachutts | 475327 | 2 | 194051 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 1 | 249978 |
| Pennsylvania | 432879 | 2 | 176722 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 2 | 257425 |
| United States | 3615920 | 18 |  |

## Priority Numbers



## Priority Numbers

$$
\begin{array}{lll}
16 & \text { VA } & 2 \\
17 & \text { MA } & 2 \\
18 & \text { PA } & 2 \\
19 & \text { VA } & 3
\end{array}
$$



## Priority Numbers

|  | Census 1790 |  |  | H-H | Huntington - Hill |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | State | Population | Seats | Priority |  |
| 16 VA 2 | Connecticut | 236841 | 1 | 167471 |  |
| 17 MA 2 | Delaware | 55540 | 1 | 39272 |  |
| 18 PA 2 | Georgia | 70835 | 1 | 50087 | $\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |
| 19 VA 3 | Kentucky | 68705 | 1 | 48581 |  |
|  | Maryland | 278514 | 1 | 196939 | $\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}$ |
|  | Massassachutts | 475327 | 2 | 194051 |  |
|  | New Hampshire | 141822 | 1 | 100283 | $\operatorname{PN}(3)=p / \sqrt{3 \times 4}=p / \sqrt{12}$ |
|  | New Jersey | 179570 | 1 | 126975 |  |
|  | New York | 331589 | 1 | 234468 |  |
|  | North Carolina | 353523 | 1 | 249978 |  |
|  | Pennsylvania | 432879 | 2 | 176722 |  |
|  | Rhode Island | 68446 | 1 | 48398 |  |
|  | South Carolina | 206236 | 1 | 145830 |  |
|  | Vermont | 85533 | 1 | 60480 |  |
|  | Virginia | 630560 | 3 | 182026 |  |
|  | United States | 3615920 | 19 |  |  |

## Priority Numbers

|  | Census 1790 |  |  | H-H | Huntington - Hill |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | State | Population | Seats | Priority |  |
| 16 VA 2 | Connecticut | 236841 | 1 | 167471 |  |
| 17 MA 2 | Delaware | 55540 | 1 | 39272 |  |
| 18 PA 2 | Georgia | 70835 | 1 | 50087 | $\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |
| 19 VA 3 | Kentucky | 68705 | 1 | 48581 |  |
|  | Maryland | 278514 | 1 | 196939 | $\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}$ |
|  | Massassachutts | 475327 | 2 | 194051 |  |
|  | New Hampshire | 141822 | 1 | 100283 | $\operatorname{PN}(3)=p / \sqrt{3 \times 4}=p / \sqrt{12}$ |
|  | New Jersey | 179570 | 1 | 126975 |  |
|  | New York | 331589 | 1 | 234468 |  |
|  | North Carolina | 353523 | 1 | 249978 |  |
|  | Pennsylvania | 432879 | 2 | 176722 |  |
|  | Rhode Island | 68446 | 1 | 48398 |  |
|  | South Carolina | 206236 | 1 | 145830 |  |
|  | Vermont | 85533 | 1 | 60480 |  |
|  | Virginia | 630560 | 3 | 182026 |  |
|  | United States | 3615920 | 19 |  |  |

## Priority Numbers



| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| Massassachutts | 475327 | 2 | 194051 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 2 | 249978 |
| Pennsylvania | 432879 | 2 | 176722 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 3 | 182026 |
| United States | 3615920 | 20 |  |

Huntington - Hill
$\operatorname{PN}(1)=p / \sqrt{1 \times 2}=p / \sqrt{2}$
$\operatorname{PN}(2)=p / \sqrt{2 \times 3}=p / \sqrt{6}$
$\operatorname{PN}(3)=p / \sqrt{3 \times 4}=p / \sqrt{12}$

## Priority Numbers



| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
| Maryland | 278514 | 1 | 196939 |
| PN(1) $=p / \sqrt{1 \times 2}=p / \sqrt{2}$ |  |  |  |
| Massassachutts | 475327 | 2 | 194051 |
| New Hampshire | 141822 | 1 | 100283 |
| New Jersey | 179570 | 1 | 126975 |
| New York | 331589 | 1 | 234468 |
| North Carolina | 353523 | 2 |  |
| Pennsylvania | 432879 | 2 | 176722 |
| Rhode Island | 68446 | 1 | 48398 |
| South Carolina | 206236 | 1 | 145830 |
| Vermont | 85533 | 1 | 60480 |
| Virginia | 630560 | 3 | 182026 |
| United States | 3615920 | 20 |  |

## Priority Numbers



| Census 1790 |  |  | H-H |
| ---: | ---: | ---: | ---: |
| State | Population | Seats | Priority |
| Connecticut | 236841 | 1 | 167471 |
| Delaware | 55540 | 1 | 39272 |
| Georgia | 70835 | 1 | 50087 |
| Kentucky | 68705 | 1 | 48581 |
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Huntington - Hill
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## The Future: Reform?

## Four Proposals:

## The Future: Reform?

## Four Proposals:

- Thirty-thousand.org
- The Wyoming Rule
- Neubauer and Gartner
- Webster's Method


## thirty-thousand.org

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CA: 1244 seats!

## thirty-thousand.org

Here's an example of a concerned group:

## http://www.thirty-thousand.org/

Thirty-thousand.org advocates 50000/representative.
This leads to a House with 6181 representatives using Webster's method of rounding.

California gets 747 seats.

## The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).

## Wyoming Rule: YouTube Video

Wikipedia
http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/

## The Wyoming Rule

Here are the results of applying the WY Rule to the 2000 and 2010 censuses.

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2000 smallest state: WY, 493782.
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2010 smallest state: WY, 563626 $h=543$ Dean HI
$h=542$ Huntington-Hill $h=540$ Webster NJ, SD

## A Proposal

A Proposal for Apportioning the House
Michael G. Neubauer, CSU Northridge, Mathematics
Margo G. (Gartner) Carr, Fordham University
...the problem of finding a "good" house size and "right" apportionment method are best considered together.

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Source: PSC 44(1), January 2011: 1—3.

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Source: PSC 44(1), January 2011: 1—3.

## Webster's Method

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The research of Balinski and Young has produced two key results. Since the Alabama paradox is a deal-breaker, then congressional apportionment must be based on a divisor method.

Further, Webster's is the only rounding method that is unbiased regarding population size.

## Related Problems

Other problems related to apportionment include:

One Voter, One Vote: The Apportionment of Congressional Seats Reconsidered Author(s): Howard A. Scarrow
Source: Polity, Vol. 22, No. 2 (Winter, 1989), pp. 253-268
Published by: Palgrave Macmillan Journals
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$>$ Decision: how does one decide the winner?

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## Thank You

It is time that I took my seat in this House!
http://www.nia977.wix.com/drbcap

## Bonus Resources

## US Census Bureau

The U.S. Census Bureau is housed within the Department of Commerce.

Check out the U.S. Census Bureau for what it says about apportionment.
http://www.census.gov/

Summary 7-page brochure:
http://www.census.gov/prod/cen2010/briefs/c2010br-08.pdf

History of Legislation:
http://www.census.gov/history/www/reference/apportionment/apportion ment legislation 1790 - 1830.html

## The Apportionment Problem

The Problem is nicely explained in the website:
http://www.ams.org/samplings/feature-column/fcarc-apportion1

## Washington's Veto

United States [Philadelphia] April 51792.

## Gentlemen of the House of Representatives

I have maturely considered the Act passed by the two Houses, intitled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.

First-The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.

Second-The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the seperate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.

George Washington.

## First Apportionment

## Act

> Chap. XXIII.-An Jet for apportioning Representatives among the several States, according to the first enumeration.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachussetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

Approved, April 14, 1792.

## US History

The first proposed amendment to the US Constitution was called Article the First, also referred to as the Congressional Apportionment Amendment:
http://en.wikipedia.org/wiki/Article the First

## More!

For playing around, learning or teaching:
http://www.cut-the-knot.org/ctk/Democracy.shtml

## Key Decades

The key decades in the history of the Congressional apportionment problem are 1790, 1840 and 1850, and 1920. Here are some excellent resources for each of these periods.
> Edmund J. James, The First Apportionment of Federal Representatives in the United States, Annals of the American Academy of Political and Social Science, 9 (January 1897): 1-41.
> Johanna Nicol Shields, Whigs Reform the "Bear Garden":
Representation and the Apportionment Act of 1842, Journal of the Early Republic, 5 (Fall 1983): 356-82.
> Charles W. Eagles, Democracy Delayed: Congressional Reapportionment and Urban-Rural Conflict in the 1920s, University of Georgia Press, 1990.

## US History

For any serious research of U.S. history, one must know about the Journals of Congress which includes the House Journal and the Senate Journal:
http://memory.loc.gov/ammem/amlaw/lwhj.html

## Presidential Elections

The 1870s saw a new twist in apportionment that spilled over into a Presidential election. In the apportionment of 1871, the House size was set to 292. Hamilton's method was legally in place. Yet the actual apportionment approved by Congress differed in four states from the Hamilton apportionment. NY was assigned 33 seats, IL 19, NH 3, and FL 2. But Hamilton's method would have given NY 34, IL 20, NH 2, and FL 1. Whatever Congress may have intended, the apportionment they approved is one that would have been given by Dean's method for the Census of 1870.

Source:
http://mathdl.maa.org/mathDL/46/?pa=content\&sa=viewDocument\&nodeld=3163\&pf=1

## Presidential Elections

Why is this such a big deal? In the closely contested election of 1876, Samuel Tilden won NY while his opponent, Rutherford B. Hayes, won the other three states. Hayes beat Tilden in the Electoral College 185 to 184. Had Hamilton's method been followed, the count in the College would have been reversed and Tilden would have been elected!

See the spreadsheet 1876 apportion for an illustration of the Hamilton calculation as compared to the actual apportionment and for a tabulation of the electoral votes in the election of 1876.

## Presidential Elections

So in 1876 , Hayes won under a Dean apportionment but would have lost under a Hamilton apportionment, even if no other factors had changed. Now let's jump forward to the Presidential election of 2000. In the Electoral College, George W. Bush defeated Al Gore by a tally of 271 to 266 . (Gore should have had 267 votes, but one of his electors from Washington, D.C. abstained.) Had the Congress used Jefferson's method to apportion the House after the 1990 census, Gore would have garnered 271 electoral votes and become the President. Even more intriguingly, had Hamilton's method been in place, the Electoral College vote would have been tied at 269 and the election thrown to the House of Representatives for resolution. Methods of apportionment do have practical consequences!

## 1790: Why 33000?

| State | Population | 30000 | 31000 | 32000 | 33000 | 34000 | 35000 | 36000 | 37000 | 38000 | 39000 | 40000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CN | 236841 | 0.8947 | 0.6400 | 0.4013 | 0.1770 | 0.9659 | 0.7669 | 0.5789 | 0.4011 | 0.2327 | 0.0728 | 0.9210 |
| DE | 55540 | 0.8513 | 0.7916 | 0.7356 | 0.6830 | 0.6335 | 0.5869 | 0.5428 | 0.5011 | 0.4616 | 0.4241 | 0.3885 |
| GA | 70835 | 0.3612 | 0.2850 | 0.2136 | 0.1465 | 0.0834 | 0.0239 | 0.9676 | 0.9145 | 0.8641 | 0.8163 | 0.7709 |
| KY | 68705 | 0.2902 | 0.2163 | 0.1470 | 0.0820 | 0.0207 | 0.9630 | 0.9085 | 0.8569 | 0.8080 | 0.7617 | 0.7176 |
| MD | 278514 | 0.2838 | 0.9843 | 0.7036 | 0.4398 | 0.1916 | 0.9575 | 0.7365 | 0.5274 | 0.3293 | 0.1414 | 0.9629 |
| MA | 475327 | 0.8442 | 0.3331 | 0.8540 | 0.4038 | 0.9802 | 0.5808 | 0.2035 | 0.8467 | 0.5086 | 0.1879 | 0.8832 |
| NH | 141822 | 0.7274 | 0.5749 | 0.4319 | 0.2976 | 0.1712 | 0.0521 | 0.9395 | 0.8330 | 0.7322 | 0.6365 | 0.5456 |
| NJ | 179570 | 0.9857 | 0.7926 | 0.6116 | 0.4415 | 0.2815 | 0.1306 | 0.9881 | 0.8532 | 0.7255 | 0.6044 | 0.4893 |
| NY | 331589 | 0.0530 | 0.6964 | 0.3622 | 0.0482 | 0.7526 | 0.4740 | 0.2108 | 0.9619 | 0.7260 | 0.5023 | 0.2897 |
| NC | 353523 | 0.7841 | 0.4040 | 0.0476 | 0.7128 | 0.3977 | 0.1007 | 0.8201 | 0.5547 | 0.3032 | 0.0647 | 0.8381 |
| PA | 432879 | 0.4293 | 0.9638 | 0.5275 | 0.1175 | 0.7317 | 0.3680 | 0.0244 | 0.6994 | 0.3916 | 0.0995 | 0.8220 |
| RI | 68446 | 0.2815 | 0.2079 | 0.1389 | 0.0741 | 0.0131 | 0.9556 | 0.9013 | 0.8499 | 0.8012 | 0.7550 | 0.7112 |
| SC | 206236 | 0.8745 | 0.6528 | 0.4449 | 0.2496 | 0.0658 | 0.8925 | 0.7288 | 0.5739 | 0.4273 | 0.2881 | 0.1559 |
| VT | 85533 | 0.8511 | 0.7591 | 0.6729 | 0.5919 | 0.5157 | 0.4438 | 0.3759 | 0.3117 | 0.2509 | 0.1932 | 0.1383 |
| VA | 630560 | 0.0187 | 0.3406 | 0.7050 | 0.1079 | 0.5459 | 0.0160 | 0.5156 | 0.0422 | 0.5937 | 0.1682 | 0.7640 |
| US | 3615920 | 8.5307 | 8.6426 | 6.9975 | 4.5733 | 6.3506 | 7.3120 | 9.4422 | 9.7276 | 8.1558 | 5.7159 | 9.3980 |
|  | Unrepresen | 255920 | 267920 | 223920 | 150920 | 215920 | 255920 | 339920 | 359920 | 309920 | 222920 | 375920 |

## Alabama Paradox

How is this possible?

| State | House $\mathbf{2 9 9}$ | House $\mathbf{3 0 0}$ |
| :--- | ---: | ---: |
| AL | 7.646 | 7.671 |
| TX | 9.640 | 9.672 |
| IL | 18.640 | 18.702 |

With the House size at 299, Alabama was the last state to be allotted an extra representative to make the House size because of it's decimal. When the House size was increased to 300, all states' quotas were increased by $0.33 \%$. And there were two states that got the extra representatives; and, this time, Texas and Illinois beat out Alabama.

## Gerrymandering


https://en.wikipedia.org/wiki/North Carolina's congressional districts

## Gerrymandering



## Illinois congressional districts.

