## The Apportionment Problem Bringing Down the House

Charles Biles, Ph.D.
Mathematics 10: Nature of Mathematics
Santa Rosa Junior College
Spring Semester 2018
website: nia977.wix.com/drbcap
"... no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, ..."

James Madison
The Federalist 55

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## The Apportionment Problem

Determine how many seats in the U.S.
House of Representatives each state gets.

## CONGRESSIONAL SEATS


(US apportionment population $=309,183,463$ )/435 $\approx 710,767$
http://www.census.gov/2010census/data/apportionment-data.php

## The First Census 1790

| State | Population |  |
| ---: | ---: | ---: |
| CT | 5 | 236841 |
| DE | 1 | 55540 |
| GA | 3 | 70835 |
| KY | 2 | 68705 |
| MD | 6 | 278514 |
| MA | 8 | 475327 |
| NH | 3 | 141822 |
| NJ | 4 | 179570 |
| NY | 6 | 331589 |
| NC | 5 | 353523 |
| PA | 8 | 432879 |
| RI | 1 | 68446 |
| SC | 5 | 206236 |
| VT | 2 | 85533 |
| VA | 10 | 630560 |
| US | 67 | 3615920 |

The first apportionment population census.

Source:
Balinski and Young, Fair Representation, Second Edition, 2001, page 158.

## Proposals

Your House size:

## Proposals

Your House size:

$$
\begin{aligned}
& h<69 \\
& h=69 \quad 1792 \text { House size } \\
& h>69 \text { and } h<100 \\
& h=100 \text { nice looking number } \\
& 100<h<105 \\
& h=105 \Leftarrow \text { remember this } \\
& 105<h<112 \\
& h=112 \Leftarrow \text { remember this } \\
& 112<h<120 \\
& h=120 \Leftarrow \text { remember this } \\
& h>120 \text { unconstitutional }
\end{aligned}
$$

## First Apportionment Bills



## First Apportionment Bills

| Census 1790 |  |  |
| :---: | ---: | ---: |
| State Population |  |  |
|  | CT | 236841 |
| DE | 55540 |  |
| GA | 70835 |  |
| KY | 68705 |  |
|  | MD | 278514 |
|  | MA | 475327 |
|  | NH | 141822 |
|  | NJ | 179570 |
|  | NY | 331589 |
|  | NC | 353523 |
|  | PA | 432879 |
|  | RI | 68446 |
|  | SC | 206236 |
|  | VT | 85533 |
|  | VA | 630560 |
|  | US | 3615920 |
|  |  |  |

3792621 - City of Los Angeles 2010

## First Apportionment Bills

|  | Census 1790 |  | House Bil |
| :---: | :---: | :---: | :---: |
|  | State | Population | 30000 |
|  | CT | 236841 |  |
|  | DE | 55540 |  |
|  | GA | 70835 |  |
|  | KY | 68705 |  |
|  | MD | 278514 |  |
|  | MA | 475327 |  |
|  | NH | 141822 |  |
|  | NJ | 179570 |  |
|  | NY | 331589 |  |
|  | NC | 353523 |  |
|  | PA | 432879 |  |
|  | RI | 68446 |  |
|  | SC | 206236 |  |
|  | VT | 85533 |  |
|  | VA | 630560 |  |
|  | US | 3615920 |  |

## First Apportionment Bills

|  | Cen | sus 1790 | House Bil |
| :---: | :---: | :---: | :---: |
|  | State | Population | Divisor 30000 |
|  | CT | 236841 |  |
|  | DE | 55540 |  |
|  | GA | 70835 |  |
|  | KY | 68705 |  |
|  | MD | 278514 |  |
|  | MA | 475327 |  |
|  | NH | 141822 |  |
|  | NJ | 179570 |  |
|  | NY | 331589 |  |
|  | NC | 353523 |  |
|  | PA | 432879 |  |
|  | RI | 68446 |  |
|  | SC | 206236 |  |
|  | VT | 85533 |  |
|  | VA | 630560 |  |
|  | US | 3615920 |  |

## First Apportionment Bills

|  | Cen | us 1790 | House Bil |
| :---: | :---: | :---: | :---: |
|  | State | Population | Divisor 30000 |
|  | CT | 236841 | 7.895 |
|  | DE | 55540 | 1.851 |
|  | GA | 70835 | 2.361 |
|  | KY | 68705 | 2.290 |
|  | MD | 278514 | 9.284 |
|  | MA | 475327 | 15.844 |
|  | NH | 141822 | 4.727 |
|  | NJ | 179570 | 5.986 |
|  | NY | 331589 | 11.053 |
|  | NC | 353523 | 11.784 |
|  | PA | 432879 | 14.429 |
|  | RI | 68446 | 2.282 |
|  | SC | 206236 | 6.875 |
|  | VT | 85533 | 2.851 |
|  | VA | 630560 | 21.019 |
|  | US | 3615920 |  |

## First Apportionment Bills

|  |  |  | House Bill |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Census 1790 <br> State Population |  | Divisor 3000 | Seats |
|  | CT | 236841 | 7.895 | 7 |
|  | DE | 55540 | 1.851 | 1 |
|  | GA | 70835 | 2.361 | 2 |
|  | KY | 68705 | 2.290 | 2 |
|  | MD | 278514 | 9.284 | 9 |
|  | MA | 475327 | 15.844 | 15 |
|  | NH | 141822 | 4.727 | 4 |
|  | NJ | 179570 | 5.986 | 5 |
|  | NY | 331589 | 11.053 | 11 |
|  | NC | 353523 | 11.784 | 11 |
|  | PA | 432879 | 14.429 | 14 |
|  | RI | 68446 | 2.282 | 2 |
|  | SC | 206236 | 6.875 | 6 |
|  | VT | 85533 | 2.851 | 2 |
|  | VA | 630560 | 21.019 | 21 |
|  | US | 3615920 |  |  |

## First Apportionment Bills



## First Apportionment Bills

Census 1790
State Population
CT 236841
DE 55540
GA 70835
KY 68705
MD 278514
MA 475327
NH 141822
NJ 179570
NY 331589
NC 353523
PA 432879
RI 68446
SC 206236
VT 85533

| VA | 630560 |
| :--- | ---: |
| US | 3615920 |

House Bill
Divisor 30000 Seats
$7.895 \quad 7$
$1.851 \quad 1$
$2.361 \quad 2$
$2.290 \quad 2$
$9.284 \quad 9$
$15.844 \quad 15$
$4.727 \quad 4$
$5.986 \quad 5$
$11.053 \quad 11$
$11.784 \quad 11$
$14.429 \quad 14$
$2.282 \quad 2$
$6.875 \quad 6$
$2.851 \quad 2$
$21.019 \quad 21$
112

Senate Bill
Divisor 33000 Seats

| 7.177 | 7 |
| ---: | ---: |
| 1.683 | 1 |
| 2.147 | 2 |
| 2.082 | 2 |
| 8.440 | 8 |
| 14.404 | 14 |
| 4.298 | 4 |
| 5.442 | 5 |
| 10.048 | 10 |
| 10.713 | 10 |
| 13.118 | 13 |
| 2.074 | 2 |
| 6.250 | 6 |
| 2.592 | 2 |
| 19.108 | 19 |

## First Apportionment Bills

Census 1790
State Population
CT 236841
DE 55540
GA 70835
KY 68705
MD 278514
MA 475327
NH 141822
NJ 179570
NY 331589
NC 353523
PA 432879
RI 68446
SC 206236
VT 85533

| VA | 630560 |
| :--- | ---: |
| US | 3615920 |

House Bill
Divisor 30000 Seats
7.8957
$1.851 \quad 1$
$2.361 \quad 2$
$2.290 \quad 2$
$9.284 \quad 9$
15.84415
$4.727 \quad 4$
$5.986 \quad 5$
$11.053 \quad 11$
$11.784 \quad 11$
$14.429 \quad 14$
$2.282 \quad 2$
$6.875 \quad 6$
$2.851 \quad 2$
$21.019 \quad 21$
112

## Rule of Three

Federalists in Congress apply a new idea:
Multiply the House size by each state's proportion to determine the state's quota (fair share of the House).

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Multiply the House size by each state's proportion to determine the state's quota (fair share of the House).

$$
q u o t a=(\text { House size }) \times \frac{\text { state population }}{\text { national population }}
$$

## Rule of Three

## Federalists in Congress apply a new idea:

Multiply the House size by each state's proportion to determine the state's quota (fair share of the House).

$$
q u o t a=(\text { House size }) \times \frac{\text { state population }}{\text { national population }}
$$

## Rule of Three

## The House Bill

Census 1790

| State |  |
| :---: | ---: |
| CT | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

22

## The House Bill

Census 1790

| State |  |
| :---: | ---: |
| CT | Population |
| DE | 536841 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

House Bill

## Divisor 30000 Seats

7.8957
$1.851 \quad 1$
$2.361 \quad 2$
$2.290 \quad 2$
$9.284 \quad 9$
15.84415
$4.727 \quad 4$
$5.986 \quad 5$
$11.053 \quad 11$
$11.784 \quad 11$
$14.429 \quad 14$
$2.282 \quad 2$
$6.875 \quad 6$
$2.851 \quad 2$
$\begin{array}{rr}21.019 \quad 21 \\ & 112\end{array}$

## The House Bill

Census 1790

| State |  |
| :---: | ---: |
| CT | 2368410 ation |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

House Bill

| Divisor $\mathbf{3 0 0 0 0}$ | Seats |
| ---: | ---: |
| 7.895 | 7 |
| 1.851 | 1 |
| 2.361 | 2 |
| 2.290 | 2 |
| 9.284 | 9 |
| 15.844 | 15 |
| 4.727 | 4 |
| 5.986 | 5 |
| 11.053 | 11 |
| 11.784 | 11 |
| 14.429 | 14 |
| 2.282 | 2 |
| 6.875 | 6 |
| 2.851 | 2 |
| 21.019 | 21 |
|  | 112 |
|  |  |

$h=112$

## The House Bill

Census 1790

| State |  |
| :---: | ---: |
| Population | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

House Bill

| Divisor $\mathbf{3 0 0 0 0}$ | Seats | Quota $\boldsymbol{h}=\mathbf{1 1 2}$ |
| :---: | :---: | ---: |
| 7.895 | 7 | 7.336 |
| 1.851 | 1 | 1.720 |
| 2.361 | 2 | 2.194 |
| 2.290 | 2 | 2.128 |
| 9.284 | 9 | 8.627 |
| 15.844 | 15 | 14.723 |
| 4.727 | 4 | 4.393 |
| 5.986 | 5 | 5.562 |
| 11.053 | 11 | 10.271 |
| 11.784 | 11 | 10.950 |
| 1.429 | 14 | 13.408 |
| 2.282 | 2 | 2.120 |
| 6.875 | 6 | 6.388 |
| 2.851 | 2 | 2.649 |
| 21.019 | 21 | 19.531 |
|  | 112 | 112 |

## Problem

| Census 1790 |  |
| :---: | ---: |
| State |  |
| Copulation | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

House Bill

| Divisor $\mathbf{3 0 0 0 0}$ | Seats | Quota $\boldsymbol{h}=\mathbf{1 1 2}$ |
| :---: | :---: | ---: |
| 7.895 | 7 | 7.336 |
| 1.851 | 1 | 1.720 |
| 2.361 | 2 | 2.194 |
| 2.290 | 2 | 2.128 |
| 9.284 | 9 | 8.627 |
| 15.844 | 15 | 14.723 |
| 4.727 | 4 | 4.393 |
| 5.986 | 5 | 5.562 |
| 11.053 | 11 | 10.271 |
| 11.784 | 11 | 10.950 |
| 14.429 | 14 | 13.408 |
| 2.282 | 2 | 2.120 |
| 6.875 | 6 | 6.388 |
| 2.851 | 2 | 2.649 |
| 21.019 | 21 | 19.531 |
|  | 112 | 112 |

The Quota Rule is violated.

## The Senate Bill

| Census 1790 |  |
| :---: | ---: |
| State |  |
| Copulation | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

Senate Bill

| Divisor $\mathbf{3 3 0 0 0}$ Seats | Quota $\boldsymbol{h}=\mathbf{1 0 5}$ |  |
| :---: | :---: | :---: |
| 7.177 | 7 | 6.877 |
| 1.683 | 1 | 1.613 |
| 2.147 | 2 | 2.057 |
| 2.082 | 2 | 1.995 |
| 8.440 | 8 | 8.088 |
| 14.404 | 14 | 13.803 |
| 4.298 | 4 | 4.118 |
| 5.442 | 5 | 5.214 |
| 10.048 | 10 | 9.629 |
| 10.713 | 10 | 10.266 |
| 13.118 | 13 | 12.570 |
| 2.074 | 2 | 1.988 |
| 6.250 | 6 | 5.989 |
| 2.592 | 2 | 2.484 |
| 19.108 | 19 | 18.310 |
|  | 105 | 105 |



No
Quota Rule Violation

## Problem

| Census 1790 |  | Senate Bill |  | Quota $\mathrm{h}=105$ |
| :---: | :---: | :---: | :---: | :---: |
| State | Population | Divisor 330 | Seats |  |
| CT | 236841 | 7.177 | 7 | 6.877 |
| DE | 55540 | 1.683 | 1 | 1.613 |
| GA | 70835 | 2.147 | 2 | 2.057 |
| KY | 68705 | 2.082 | 2 | 1.995 |
| MD | 278514 | 8.440 | 8 | 8.088 |
| MA | 475327 | 14.404 | 14 | 13.803 |
| NH | 141822 | 4.298 | 4 | 4.118 |
| NJ | 179570 | 5.442 | 5 | 5.214 |
| NY | 331589 | 10.048 | 10 | 9.629 |
| NC | 353523 | 10.713 | 10 | 10.266 |
| PA | 432879 | 13.118 | 13 | 12.570 |
| RI | 68446 | 2.074 | 2 | 1.988 |
| SC | 206236 | 6.250 | 6 | 5.989 |
| VT | 85533 | 2.592 | 2 | 2.484 |
| VA | 630560 | 19.108 | 19 | 18.310 |
| US | 3615920 |  | 105 | 105 |

Large states are favored over small states.

## Hamilton's Method

| State | Population |  |
| :---: | ---: | ---: |
| CT | 236841 |  |
| DE | 55540 |  |
| GA | 70835 |  |
| KY | 68705 |  |
| MD | 278514 |  |
| MA | 475327 |  |
| NH | 141822 |  |
| NJ | 179570 |  |
| NY | 331589 |  |
| NC | 353523 |  |
| PA | 432879 |  |
| RI | 68446 |  |
| SC | 206236 |  |
| VT | 85533 |  |
| VA | 630560 |  |
| US | 3615920 | 120.5307 |

## Hamilton's Method

| State | Population | $h=120$ |
| :---: | :---: | :---: |
| CT | 236841 |  |
| DE | 55540 |  |
| GA | 70835 |  |
| KY | 68705 |  |
| MD | 278514 |  |
| MA | 475327 |  |
| NH | 141822 |  |
| NJ | 179570 |  |
| NY | 331589 |  |
| NC | 353523 |  |
| PA | 432879 |  |
| RI | 68446 |  |
| SC | 206236 |  |
| VT | 85533 |  |
| VA | 630560 |  |
| US | 3615920 | 120.5307 |
|  |  | $=30000$ |

## Hamilton's Method



## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q |
| :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 |
| DE | 55540 |  | 1.843 | 1 |
| GA | 70835 |  | 2.351 | 2 |
| KY | 68705 |  | 2.280 | 2 |
| MD | 278514 |  | 9.243 | 9 |
| MA | 475327 |  | 15.774 | 15 |
| NH | 141822 |  | 4.707 | 4 |
| NJ | 179570 |  | 5.959 | 5 |
| NY | 331589 |  | 11.004 | 11 |
| NC | 353523 |  | 11.732 | 11 |
| PA | 432879 |  | 14.366 | 14 |
| RI | 68446 |  | 2.271 | 2 |
| SC | 206236 |  | 6.844 | 6 |
| VT | 85533 |  | 2.839 | 2 |
| VA | 630560 |  | 20.926 | 20 |
| US | 3615920 | 120.5307 | 120 | 111 |
| $d=30000$ |  |  |  |  |

32

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |
| $d=30000$ |  |  |  |  |  |

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

This became the first apportionment bill passed by Congress.

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

This became the first apportionment bill passed by Congress.

The bill is sent to President Washington for approval.

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

This became the first apportionment bill passed by Congress.

The bill is sent to President Washington for approval.

Washington vetoes the bill.

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

[^0]
## Hamilton's Method

| State | Population | $h=120$ | Quota Lower Q | Appt |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.8607 | 8 | Connecticut: |
| DE | 55540 |  | 1.843 1 | 2 | 236841/8 = 29605.13 |
| GA | 70835 |  | $2.351 \quad 2$ | 2 |  |
| KY | 68705 |  | 2.280 | 2 |  |
| MD | 278514 |  | 9.243 9 | 9 | Delaware: |
| MA | 475327 |  | 15.77415 | 16 | 55540/2 = 27770 |
| NH | 141822 |  | 4.7074 | 5 |  |
| NJ | 179570 |  | 5.959 | 6 |  |
| NY | 331589 |  | 11.00411 | 11 |  |
| NC | 353523 |  | 11.73211 | 12 |  |
| PA | 432879 |  | 14.36614 | 14 |  |
| RI | 68446 |  | 2.271 | 2 |  |
| SC | 206236 |  | $6.844 \quad 6$ | 7 |  |
| VT | 85533 |  | 2.8392 | 3 |  |
| VA | 630560 |  | $20.926 \quad 20$ | 21 | U.S.: |
| US | 3615920 | 120.5307 | $120 \quad 111$ | 120 | $3615920 / 120=30,132.66 \ldots$ |

## Basic Jefferson Method

After Washington's veto letter of 5 April 1792, Congress quickly passes the original Senate bill. Washington signed the bill on 14 April 1792.

# Two Methodologies 

- Divisor Methods
- Quota Methods


## Two Methodologies

- Divisor Methods
- Basic
- Modified
- Quota Methods


## Two Methodologies

- Divisor Methods
- Basic: $h$ is the result
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## Two Methodologies

- Divisor Methods
- Basic: $h$ is the result
- Modified: $h$ is the goal
- Quota Methods
$h$ is the resource


## Basic Jefferson Method

1. Decide on a divisor $d$ (constituency).

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2. Calculate each state's quotient:

$$
\begin{aligned}
\text { quotient } & =\text { population/divisor } \\
q & =p / d
\end{aligned}
$$

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\end{aligned}
$$

3. The state's apportionment is the integer part of $q$ : $a=\operatorname{int}(q)$.

The resulting house size is the sum of each state's apportionment.

## First 60 years

A Basic Divisor Method would be used as the House apportionment method until 1850.


## Basic Jefferson Method

Problems are discovered as the method is used; however, defects of the method were evident from the beginning.


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Jefferson's method systematically favors larger states; further, it can violate the Quota Rule.

## 1830 Census

Three new methods are proposed to deal with the decimal part of a state's quotient.

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## 1830 Census

Three new methods are proposed to deal with the decimal part of a state's quotient.

Jefferson: round down (drop the decimal).
Adams: round up.
Dean: round down or up according to which option gives a state's constituency closest to the divisor.

Webster: round normally.


## James Dean

In 1830 the US population was 11,931,578. Consider: constituency = 50,000 people.

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Vermont's population: 280,657.
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With 5 seats the constituency is $280,657 / 5=56,131$.
With 6 seats the constituency is $280,657 / 6=46,776$.

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At this point, Jefferson apportions 5 seats to Vermont; Adams, 6 seats.

With 5 seats the constituency is 280,657/5 = 56,131.
With 6 seats the constituency is $280,657 / 6=46,776$.

A constituency of 46,776 is closer to the target constituency of 50,000; hence, Dean awards Vermont 6 seats.

## James Dean

Step 1: Select the constituency, $d$.
Step 2: Calculate $q=p / d$ and $n=\operatorname{int}(q)$.
Step 3: Let the apportionment be either $n$ or $n+1$, with $n+1$ iff $p /(n+1)$ is closer to $d$ than $p / n$.

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This is mathematically equivalent to: let the apportionment be $n+1$ iff $q \geq \operatorname{HM}(n, n+1)$.

## Daniel Webster

Step 1: Select the constituency, $d$.
Step 2: Calculate $q=p / d$ and $n=\operatorname{int}(q)$.
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## Daniel Webster

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Dean:


Webster:


## Dean and Webster

Step 1: Select the constituency, $d$.
Step 2: Calculate $q=p / d$ and $n=\operatorname{int}(q)$.
Step 3: Let the apportionment be either $n$ or $n+1$, with $n+1$ if and only if

Dean:


$$
\text { Dean: } a=n+1 \Leftrightarrow H M(n, n+1) \leq q \text {. }
$$

Webster:


Webster: $a=n+1 \Leftrightarrow \mathrm{AM}(n, n+1) \leq q$.

## 1830 Census

In 1831 there were four different proposed apportionment methods based on a given divisor. The difference was in how the method chose to round a state's quotient (state's population divided by the chosen divisor).

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In 1831 there were four different proposed apportionment methods based on a given divisor. The difference was in how the method chose to round a state's quotient (state's population divided by the chosen divisor).

Jefferson: round down (min).
Adams: round up (max).
Dean: round by closest constituency (HM). Webster: round normally (AM).

## In a Round About Way

| Census 1810 |  | $d=35000$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | Population | Quotient | min | AM | HM | max |
| CT | 261818 | 7.4805 | 7 | 7 | 8 | 8 |
| DE | 71004 | 2.0287 | 2 | 2 | 2 | 3 |
| GA | 210346 | 6.0099 | 6 | 6 | 6 | 7 |
| KY | 374287 | 10.6939 | 10 | 11 | 11 | 11 |
| MD | 335946 | 9.5985 | 9 | 10 | 10 | 10 |
| MA | 700745 | 20.0213 | 20 | 20 | 20 | 21 |
| NH | 214460 | 6.1274 | 6 | 6 | 6 | 7 |
| NJ | 241222 | 6.8921 | 6 | 7 | 7 | 7 |
| NY | 953043 | 27.2298 | 27 | 27 | 27 | 28 |
| NC | 487971 | 13.9420 | 13 | 14 | 14 | 14 |
| OH | 230760 | 6.5931 | 6 | 7 | 7 | 7 |
| PA | 809773 | 23.1364 | 23 | 23 | 23 | 24 |
| RI | 76931 | 2.1980 | 2 | 2 | 2 | 3 |
| SC | 336569 | 9.6163 | 9 | 10 | 10 | 10 |
| TN | 243913 | 6.9689 | 6 | 7 | 7 | 7 |
| VT | 217895 | 6.2256 | 6 | 6 | 6 | 7 |
| VA | 817594 | 23.3598 | 23 | 23 | 23 | 24 |
| US | 6575234 | 188.1222 | 181 | 188 | 189 | 198 |

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## In a Round About Way

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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75

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| Census 1810 |  | $d=35000$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | Population | Quotient | Jefferson | AM | HM | max |
| CT | 261818 | 7.4805 | 7 | 7 | 8 | 8 |
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| State | Population | Quotient | Jefferson | AM | HM | Adams |
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| Census 1810 |  | $d=35000$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | Population | Quotient | Jefferson | Webster | HM | Adams |
| CT | 261818 | 7.4805 | 7 | 7 | 8 | 8 |
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| Census 1810 |  | $d=35000$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | Population | Quotient | Jefferson | Webster | Dean | Adams |
| CT | 261818 | 7.4805 | 7 | 7 | 8 | 8 |
| DE | 71004 | 2.0287 | 2 | 2 | 2 | 3 |
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## In a Round About Way

| $\begin{aligned} & \operatorname{HM}(7,8)= \\ & 7.4666 \cdots \end{aligned}$ | Census 1810 |  | $d=35000$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | State | Population | Quotient | Jefferson | Webster | Dean | Adams |
|  | CT | 261818 | 7.4805 | 7 | 7 | 8 | 8 |
|  | DE | 71004 | 2.0287 | 2 | 2 | 2 | 3 |
|  | GA | 210346 | 6.0099 | 6 | 6 | 6 | 7 |
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## 1840 Census

In 1842 the apportionment debate began with the political game: divisor! On one day in the 242 member House, 59 motions were made to establish a divisor. The values ranged from 30000 to 141000 with majority from 50159 to 62172.

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The Apportionment Act of 1842 used a basic divisor method with $d=70680$ and Webster's method of rounding. This yielded $h=223$, the only time in U.S. history that $h$ decreased as a result of a census-based re-apportionment.

## The Vinton Act

The Vinton Act of 1850 (Representative Samuel Vinton, Whig-Ohio) was passed to head off politicizing the census figures. The idea was to adopt a permanent appropriation act.


## The Vinton Act

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But experience exposed problems with the Vinton Act.

## Lessons from History

The quota method is subject to counter-intuitive paradoxes:
$>$ The Alabama Paradox
> The Population Growth Paradox

## Alabama Paradox

The Alabama paradox may occur when applying the Hamilton quota method:
when the number of House seats is increased, a given state's apportion may decrease.

## The Deal Breaker

Results from the 1890 census doomed Hamilton's Method.


## 1910

Apportionment based on the 1910 census came from another mutation in apportionment methodology.

Congress abandoned the Quota Method and used a modified divisor method.

# Modified Divisor Methods 

Step 1. Select the House size, $h$.
Step 2. Apply a Basic Divisor Method to obtain $h$ seats.

1910: $h=433$ and Webster's method.

## 1920 Census

In the 1920 decade, for the only time in U. S. History, no census-based re-apportionment act was passed.

Congress could not agree on either the size of the House or on the method of apportionment. Further, the politics of prohibition played a significant role: the dries would not support any proposal that gave the wets more power.

## Today

The current method consists of the Census Act of 1929 (which froze $h=435$ ) and a 1941 amendment that specifies the apportionment method of

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## Today

The Huntington-Hill method is a divisor method:

$$
\text { Let } q=p / d \text { and } n=\operatorname{int}(q) \text {. }
$$

Then $a=n+1$ iff $q \geq$

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Criterion: $a=n+1$ iff $\frac{d}{\left(\frac{p}{n+1}\right)} \leq \frac{\left(\frac{p}{n}\right)}{d}$

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Dean:


H-H:


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iff $\mathrm{GM}(n, n+1) \leq q$.

## The Aftermath

Michel Balinski, Professor of Mathematics at SUNY Stony Brook and H. Peyton Young, Professor of Mathematics at Johns Hopkins, proved the following theorem in 1982:

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There are no perfect apportionment methods.

Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

## They Mean Well

A modified divisor method first fixes the House size, then seeks a divisor that when the state's quotients are rounded appropriately and summed, the house size is achieved.

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## Montana

In the 1990 apportionment Montana lost one of its two seats it held for 80 years. In 1991 MT filed suit in federal district court (MT vs. US Dept Commerce).

MT argued the H-H method is unconstitutional and that either Dean's or Adams's method should be used. The federal judges voted 2-1 in favor of MT.

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# Distributing 435 seats among 50 states according to their respective populations is a math problem. 

## Today

# Today the Census Bureau obtains apportionments using a priority technique of calculation rather than an ad-hoc technique of calculation. 

Census Bureau video 2:09 minutes. The Amazing Apportionment Machine

## Apportionment by Priority

Step 1. Give one seat to each state.
Step 2. Attach a priority number to each state.
Step 3. Award seats one at a time by priority until the desired House size is reached.

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$$
\begin{aligned}
& \text { Priority number } \\
& \text { for a state with } \\
& n \text { seats }
\end{aligned}=\frac{\text { state population }}{\text { ave }(n, n+1)}
$$



## Five Averages

## ave

- Greatest Divisors max
- Harmonic Means HaM
- Equal Proportions GeM
- Major Fractions AM
- Smallest Divisors min


## The Last Seat

## Who got the $435^{\text {th }}$ seat?

## ave 435

- Greatest Divisors max IL
- Harmonic Means HaM MN
- Equal Proportions GeM MN
- Major Fractions AM NC
- Smallest Divisors min WA


## The Last Seat

Who gets the $436^{\text {th }}$ seat?
ave $435 \quad 436$

- Greatest Divisors max IL WA
- Harmonic Means HaM MN CA
- Equal Proportions GeM MN NC
- Major Fractions AM NC MO
- Smallest Divisors min WA PA


## The Last Seat

Who gets the $436^{\text {th }}$ seat?

$$
\text { ave } 435 \quad 436
$$

- Greatest Divisors max IL WA
- Harmonic Means HaM MN CA
- Equal Proportions GeM MN NC
- Major Fractions AM NC MO
- Smallest Divisors min WA PA

Priority list based on the 2010 census using the method of Equal Proportions.

## The Future: Reform?

## Four Proposals:

## The Future: Reform?

## Four Proposals:

- Thirty-thousand.org
- Wyoming Rule
- Neubauer and Gartner
- Webster's Method.


## thirty-thousand.org

Here's an example of a concerned group:
http://www.thirty-thousand.org/

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Jefferson basic divisor method.
CA: 1244 seats!

## thirty-thousand.org

Here's an example of a concerned group:

## http://www.thirty-thousand.org/

Thirty-thousand.org advocates 50000/representative.
This creates a House with 6181 representatives using Webster's method of rounding.

California gets 747 seats.

## The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).
http://en.wikipedia.org/wiki/Wyoming Rule
http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/

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Results of applying the WY Rule to the 2000 and 2010 censuses.

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2000 smallest state: WY, 493782.
$h=569$ Huntington-Hill

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2000 smallest state: WY, 493782.
$h=569$ Huntington-Hill
2010 smallest state: WY, 563626 $h=543$ Dean HI
$h=542$ Huntington-Hill $h=540$ Webster NJ, SD

## A Proposal

A Proposal for Apportioning the House
Michael G. Neubauer, CSU Northridge, Mathematics Margo G. (Gartner) Carr, Cerro Coso Community College
...the problem of finding a "good" house size and "right" apportionment method are best considered together.

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Source: PSC 44(1), January 2011: 1—3.

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Source: PSC 44(1), January 2011: 1—3.

## Webster's Method

The simplest reform would be to replace the geometric mean of decimal rounding in the Huntington-Hill method by the arithmetic mean of decimal rounding in Webster's method.

## Webster's Method

The simplest reform would be to replace the geometric mean of decimal rounding in the Huntington-Hill method by the arithmetic mean of decimal rounding in Webster's method.

The research of Balinski and Young has produced the following two key results. Since the Alabama paradox is a deal-breaker, then congressional apportionment must be based on a divisor method. Further, Webster's is the only rounding method that is unbiased towards either larger or smaller states.


Charles M. Biles, Ph.D.
Congressional
Apportionment
HONE
віо
RESOURCES
CONTACT
Constitutional Congregressional Apportionment Problem
How many seats in the U. S. House of Representatives does each stateger?
An answer is presented as an historical narrative with relevant and timely applications in an upcoming book, The History of Congressional Apportionment.

Chapter 1. Congressional Apportionment Based on the Census: 1790.
Chapter 2. Congressional Apportionment Based on the Census: 1800-1840.
Chapter 3. Congressional Apportionment Based on the Census: 1850-1890.
Chapter 4. Congressional Apportionment Based on the Census: 1900-1930.
Chapter 5. Congressional Apportionment Based on the Census: 1940-2010.
Chapter 6. An Historical Overture.


UNDER CONSTRUCTION
Charles Biles
Theriistory of Congressional Apportionment
HSU Press
Last update: 13 June 2017.
Cover Graphic courtesy of
The West Virginia Record

This site was created using WIX.com. Create your own for FREE >>

## Thank You

It is time that I took my seat in this House!
http://www.nia977.wix.com/drbcap

## Bonus Resources

## Related Problems

Other problems related to apportionment include:

One Voter, One Vote: The Apportionment of Congressional Seats Reconsidered Author(s): Howard A. Scarrow
Source: Polity, Vol. 22, No. 2 (Winter, 1989), pp. 253-268
Published by: Palgrave Macmillan Journals
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## Gerrymandering



## Gerrymandering

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| $\bullet$ | $\bullet$ | $\bullet$ | $\bullet$ | $\bullet$ | $\bullet$ | $\bullet$ |
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Apportion into five districts.

## Gerrymandering

\section*{$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \quad 0$ $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$ <br> $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \quad 0$ <br> |  |  |  | - |  |  |  |  |  |  |  |  |  |  |  |  |
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## Gerrymandering



## Gerrymandering

$$
\begin{array}{|llllllllll|}
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\hline
\end{array}
$$

## Related Problems

Other problems related to apportionment include:
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$>$ Voting: the mechanism of voting.

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## Washington's Veto

United States [Philadelphia] April 51792.

## Gentlemen of the House of Representatives

I have maturely considered the Act passed by the two Houses, intitled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.

First-The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.

Second-The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the seperate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.

George Washington.

Copy, DNA: RG 233, Second Congress, 1791-1793, Records of Legislative Proceedings, Journals; LB, DLC:GW. (from Philander Chase, et al., eds., The Papers of George Washington, Presidential Series, Vol. 10: March-August 1792 [Charlottesville, Va., 002], 213-14).

## First Apportionment

## Act

> Chap. XXIII.-An Jet for apportioning Representatives among the scveral States, according to the first enumeration.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachussetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

Approved, April 14, 1792.

## 1790: Why 33000?

| State | Population | 30000 | 31000 | 32000 | 33000 | 34000 | 35000 | 36000 | 37000 | 38000 | 39000 | 40000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CN | 236841 | 0.8947 | 0.6400 | 0.4013 | 0.1770 | 0.9659 | 0.7669 | 0.5789 | 0.4011 | 0.2327 | 0.0728 | 0.9210 |
| DE | 55540 | 0.8513 | 0.7916 | 0.7356 | 0.6830 | 0.6335 | 0.5869 | 0.5428 | 0.5011 | 0.4616 | 0.4241 | 0.3885 |
| GA | 70835 | 0.3612 | 0.2850 | 0.2136 | 0.1465 | 0.0834 | 0.0239 | 0.9676 | 0.9145 | 0.8641 | 0.8163 | 0.7709 |
| KY | 68705 | 0.2902 | 0.2163 | 0.1470 | 0.0820 | 0.0207 | 0.9630 | 0.9085 | 0.8569 | 0.8080 | 0.7617 | 0.7176 |
| MD | 278514 | 0.2838 | 0.9843 | 0.7036 | 0.4398 | 0.1916 | 0.9575 | 0.7365 | 0.5274 | 0.3293 | 0.1414 | 0.9629 |
| MA | 475327 | 0.8442 | 0.3331 | 0.8540 | 0.4038 | 0.9802 | 0.5808 | 0.2035 | 0.8467 | 0.5086 | 0.1879 | 0.8832 |
| NH | 141822 | 0.7274 | 0.5749 | 0.4319 | 0.2976 | 0.1712 | 0.0521 | 0.9395 | 0.8330 | 0.7322 | 0.6365 | 0.5456 |
| NJ | 179570 | 0.9857 | 0.7926 | 0.6116 | 0.4415 | 0.2815 | 0.1306 | 0.9881 | 0.8532 | 0.7255 | 0.6044 | 0.4893 |
| NY | 331589 | 0.0530 | 0.6964 | 0.3622 | 0.0482 | 0.7526 | 0.4740 | 0.2108 | 0.9619 | 0.7260 | 0.5023 | 0.2897 |
| NC | 353523 | 0.7841 | 0.4040 | 0.0476 | 0.7128 | 0.3977 | 0.1007 | 0.8201 | 0.5547 | 0.3032 | 0.0647 | 0.8381 |
| PA | 432879 | 0.4293 | 0.9638 | 0.5275 | 0.1175 | 0.7317 | 0.3680 | 0.0244 | 0.6994 | 0.3916 | 0.0995 | 0.8220 |
| RI | 68446 | 0.2815 | 0.2079 | 0.1389 | 0.0741 | 0.0131 | 0.9556 | 0.9013 | 0.8499 | 0.8012 | 0.7550 | 0.7112 |
| SC | 206236 | 0.8745 | 0.6528 | 0.4449 | 0.2496 | 0.0658 | 0.8925 | 0.7288 | 0.5739 | 0.4273 | 0.2881 | 0.1559 |
| VT | 85533 | 0.8511 | 0.7591 | 0.6729 | 0.5919 | 0.5157 | 0.4438 | 0.3759 | 0.3117 | 0.2509 | 0.1932 | 0.1383 |
| VA | 630560 | 0.0187 | 0.3406 | 0.7050 | 0.1079 | 0.5459 | 0.0160 | 0.5156 | 0.0422 | 0.5937 | 0.1682 | 0.7640 |
| US | 3615920 | 8.5307 | 8.6426 | 6.9975 | 4.5733 | 6.3506 | 7.3120 | 9.4422 | 9.7276 | 8.1558 | 5.7159 | 9.3980 |
|  | Unrepresen | 255920 | 267920 | 223920 | 150920 | 215920 | 255920 | 339920 | 359920 | 309920 | 222920 | 375920 |

## Alabama Paradox

How is this possible?

| State | House $\mathbf{2 9 9}$ | House $\mathbf{3 0 0}$ |
| :--- | ---: | ---: |
| AL | 7.646 | 7.671 |
| TX | 9.640 | 9.672 |
| IL | 18.640 | 18.702 |

With the House size at 299, Alabama was the last state to be allotted an extra representative to make the House size because of it's decimal. When the House size was increased to 300, all states' quotas were increased by $0.33 \%$. And there were two states that got the extra representatives; and, this time, Texas and Illinois beat out Alabama.

## US Census Bureau

The U.S. Census Bureau is housed within the Department of Commerce.

Check out the U.S. Census Bureau for what it says about apportionment.
http://www.census.gov/

Summary 7-page brochure:
http://www.census.gov/prod/cen2010/briefs/c2010br-08.pdf

History of Legislation:
http://www.census.gov/history/www/reference/apportionment/apportion ment legislation 1790 - 1830.html

## More!

For playing around, learning or teaching:
http://www.cut-the-knot.org/ctk/Democracy.shtml

## Key Decades

The key decades in the history of the Congressional apportionment problem are 1790, 1840 and 1850, and 1920. Here are some excellent resources for each of these periods.
$>$ Edmund J. James, The First Apportionment of Federal Representatives in the United States, Annals of the American Academy of Political and Social Science, 9 (January 1897): 1-41.
> Johanna Nicol Shields, Whigs Reform the "Bear Garden":
Representation and the Apportionment Act of 1842, Journal of the Early Republic, 5 (Fall 1983): 356-82.
> Charles W. Eagles, Democracy Delayed: Congressional Reapportionment and Urban-Rural Conflict in the 1920s, University of Georgia Press, 1990.

## US History

For any serious research of U.S. history, one must know about the Journals of Congress which includes the House Journal and the Senate Journal:
http://memory.loc.gov/ammem/amlaw/lwhj.html


[^0]:    U.S.:

    3615920/120 = 30,132.66...

