The Apportionment Problem

Bringing Down the House

Charles Biles, Ph.D.
Mathematics 10: Nature of Mathematics
Santa Rosa Junior College
Fall Semester 2018

website: nia977.wix.com/drbcap

> "… no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, …"
>
> James Madison
> *The Federalist 55*

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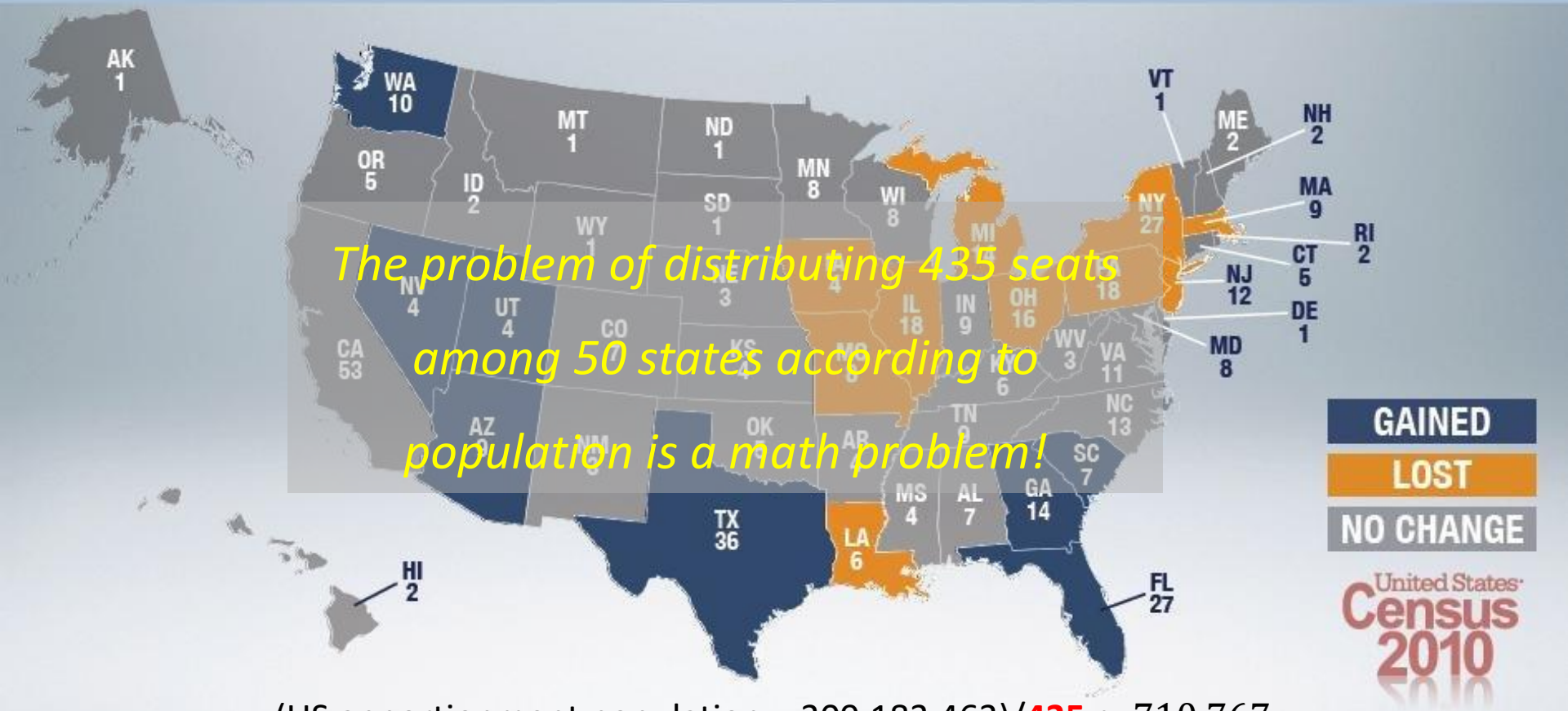
The Apportionment Problem

Determine how many seats in the U.S. House of Representatives each state gets.

CONGRESSIONAL SEATS

2010 OFFICIAL RESULTS



The problem of distributing 435 seats among 50 states according to population is a math problem!

(US apportionment population = 309,183,463)/**435** ≈ 710,767

http://www.census.gov/2010census/data/apportionment-data.php

The First Census 1790

State		Population
CT	5	236841
DE	1	55540
GA	3	70835
KY	2	68705
MD	6	278514
MA	8	475327
NH	3	141822
NJ	4	179570
NY	6	331589
NC	5	353523
PA	8	432879
RI	1	68446
SC	5	206236
VT	2	85533
VA	10	630560
US	67	3615920

The first apportionment population census.

Source:
Balinski and Young, *Fair Representation,* Second Edition, 2001, page 158.

Proposals

Your House size:

Proposals

Your House size:

$15 \le h < 69$

$h = 69$ 1792 House size

$h > 69$ and $h < 100$

$h = 100$ nice looking number

$100 < h < 105$

$h = 105$ $\Leftarrow$ remember this

$105 < h < 112$

$h = 112$ $\Leftarrow$ remember this

$112 < h < 120$

$h = 120$ $\Leftarrow$ remember this

$h > 120$ unconstitutional

First Apportionment Bills

Census 1790

State	Population
CT	236841
DE	55540
GA	70835
KY	68705
MD	278514
MA	475327
NH	141822
NJ	179570
NY	331589
NC	353523
PA	432879
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SC	206236
VT	85533
VA	630560
US	3615920

First Apportionment Bills

Census 1790

State	Population
CT	236841
DE	55540
GA	70835
KY	68705
MD	278514
MA	475327
NH	141822
NJ	179570
NY	331589
NC	353523
PA	432879
RI	68446
SC	206236
VT	85533
VA	630560
US	3615920

3792621 — City of Los Angeles 2010

First Apportionment Bills

Census 1790		House Bill
State	**Population**	**30000**
CT	236841	
DE	55540	
GA	70835	
KY	68705	
MD	278514	
MA	475327	
NH	141822	
NJ	179570	
NY	331589	
NC	353523	
PA	432879	
RI	68446	
SC	206236	
VT	85533	
VA	630560	
US	3615920	

First Apportionment Bills

Census 1790		House Bill
State	**Population**	**Divisor 30000**
CT	236841	
DE	55540	
GA	70835	
KY	68705	
MD	278514	
MA	475327	
NH	141822	
NJ	179570	
NY	331589	
NC	353523	
PA	432879	
RI	68446	
SC	206236	
VT	85533	
VA	630560	
US	3615920	

First Apportionment Bills

Census 1790		House Bill
State	**Population**	**Divisor 30000**
CT	236841	7.895
DE	55540	1.851
GA	70835	2.361
KY	68705	2.290
MD	278514	9.284
MA	475327	15.844
NH	141822	4.727
NJ	179570	5.986
NY	331589	11.053
NC	353523	11.784
PA	432879	14.429
RI	68446	2.282
SC	206236	6.875
VT	85533	2.851
VA	630560	21.019
US	3615920	

First Apportionment Bills

Census 1790		House Bill	
State	**Population**	**Divisor 30000**	**Seats**
CT	236841	7.895	7
DE	55540	1.851	1
GA	70835	2.361	2
KY	68705	2.290	2
MD	278514	9.284	9
MA	475327	15.844	15
NH	141822	4.727	4
NJ	179570	5.986	5
NY	331589	11.053	11
NC	353523	11.784	11
PA	432879	14.429	14
RI	68446	2.282	2
SC	206236	6.875	6
VT	85533	2.851	2
VA	630560	21.019	21
US	3615920		

First Apportionment Bills

Census 1790		House Bill	
State	**Population**	**Divisor 30000**	**Seats**
CT	236841	7.895	7
DE	55540	1.851	1
GA	70835	2.361	2
KY	68705	2.290	2
MD	278514	9.284	9
MA	475327	15.844	15
NH	141822	4.727	4
NJ	179570	5.986	5
NY	331589	11.053	11
NC	353523	11.784	11
PA	432879	14.429	14
RI	68446	2.282	2
SC	206236	6.875	6
VT	85533	2.851	2
VA	630560	21.019	21
US	3615920		**112**

First Apportionment Bills

Census 1790		House Bill		Senate Bill	
State	Population	Divisor 30000	Seats	Divisor 33000	Seats
CT	236841	7.895	7	7.177	7
DE	55540	1.851	1	1.683	1
GA	70835	2.361	2	2.147	2
KY	68705	2.290	2	2.082	2
MD	278514	9.284	9	8.440	8
MA	475327	15.844	15	14.404	14
NH	141822	4.727	4	4.298	4
NJ	179570	5.986	5	5.442	5
NY	331589	11.053	11	10.048	10
NC	353523	11.784	11	10.713	10
PA	432879	14.429	14	13.118	13
RI	68446	2.282	2	2.074	2
SC	206236	6.875	6	6.250	6
VT	85533	2.851	2	2.592	2
VA	630560	21.019	21	19.108	19
US	3615920		112		

First Apportionment Bills

Census 1790		House Bill		Senate Bill	
State	Population	Divisor 30000	Seats	Divisor 33000	Seats
CT	236841	7.895	7	7.177	7
DE	55540	1.851	1	1.683	1
GA	70835	2.361	2	2.147	2
KY	68705	2.290	2	2.082	2
MD	278514	9.284	9	8.440	8
MA	475327	15.844	15	14.404	14
NH	141822	4.727	4	4.298	4
NJ	179570	5.986	5	5.442	5
NY	331589	11.053	11	10.048	10
NC	353523	11.784	11	10.713	10
PA	432879	14.429	14	13.118	13
RI	68446	2.282	2	2.074	2
SC	206236	6.875	6	6.250	6
VT	85533	2.851	2	2.592	2
VA	630560	21.019	21	19.108	19
US	3615920		112		105

Rule of Three

Federalists in Congress apply a new idea:

Multiply the House size by each state's proportion to determine each state's **quota** (fair share of the House).

Rule of Three

Federalists in Congress apply a new idea:

Multiply the House size by each state's proportion to determine each state's **quota** (fair share of the House).

$$\textbf{\textit{quota}} = (House\ size) \times \frac{state\ population}{national\ population}$$

Rule of Three

Federalists in Congress apply a new idea:

Multiply the House size by each state's proportion to determine each state's **quota** (fair share of the House).

$$\textbf{\textit{quota}} = (House\ size) \times \underbrace{\frac{state\ population}{national\ population}}_{Rule\ of\ Three}$$

The House Bill

Census 1790		House Bill	
State	**Population**	**Divisor 30000**	**Seats**
CT	236841	7.895	7
DE	55540	1.851	1
GA	70835	2.361	2
KY	68705	2.290	2
MD	278514	9.284	9
MA	475327	15.844	15
NH	141822	4.727	4
NJ	179570	5.986	5
NY	331589	11.053	11
NC	353523	11.784	11
PA	432879	14.429	14
RI	68446	2.282	2
SC	206236	6.875	6
VT	85533	2.851	2
VA	630560	21.019	21
US	3615920		

The House Bill

Census 1790		House Bill	
State	Population	Divisor 30000	Seats
CT	236841	7.895	7
DE	55540	1.851	1
GA	70835	2.361	2
KY	68705	2.290	2
MD	278514	9.284	9
MA	475327	15.844	15
NH	141822	4.727	4
NJ	179570	5.986	5
NY	331589	11.053	11
NC	353523	11.784	11
PA	432879	14.429	14
RI	68446	2.282	2
SC	206236	6.875	6
VT	85533	2.851	2
VA	630560	21.019	21
US	3615920		**112**

The House Bill

Census 1790		House Bill		
State	**Population**	**Divisor 30000**	**Seats**	***h*=112**
CT	236841	7.895	7	
DE	55540	1.851	1	
GA	70835	2.361	2	
KY	68705	2.290	2	
MD	278514	9.284	9	
MA	475327	15.844	15	
NH	141822	4.727	4	
NJ	179570	5.986	5	
NY	331589	11.053	11	
NC	353523	11.784	11	
PA	432879	14.429	14	
RI	68446	2.282	2	
SC	206236	6.875	6	
VT	85533	2.851	2	
VA	630560	21.019	21	
US	3615920		**112**	

The House Bill

Census 1790		House Bill			
State	Population	Divisor 30000	Seats	Quota	*h*=112
CT	236841	7.895	7		7.336
DE	55540	1.851	1		1.720
GA	70835	2.361	2		2.194
KY	68705	2.290	2		2.128
MD	278514	9.284	9		8.627
MA	475327	15.844	15		14.723
NH	141822	4.727	4		4.393
NJ	179570	5.986	5		5.562
NY	331589	11.053	11		10.271
NC	353523	11.784	11		10.950
PA	432879	14.429	14		13.408
RI	68446	2.282	2		2.120
SC	206236	6.875	6		6.388
VT	85533	2.851	2		2.649
VA	630560	21.019	21		19.531
US	3615920		**112**		112

Problem

Census 1790		House Bill		
State	Population	Divisor 30000	Seats	Quota *h*=112
CT	236841	7.895	7	7.336
DE	55540	1.851	1	1.720
GA	70835	2.361	2	2.194
KY	68705	2.290	2	2.128
MD	278514	9.284	9	8.627
MA	475327	15.844	15	14.723
NH	141822	4.727	4	4.393
NJ	179570	5.986	5	5.562
NY	331589	11.053	11	10.271
NC	353523	11.784	11	10.950
PA	432879	14.429	14	13.408
RI	68446	2.282	2	2.120
SC	206236	6.875	6	6.388
VT	85533	2.851	2	2.649
VA	630560	21.019	21	19.531
US	3615920		112	112

The Quota Rule is violated.

The Senate Bill

Census 1790		Senate Bill		
State	Population	Divisor 33000	Seats	Quota *h*=105
CT	236841	7.177	7	6.877
DE	55540	1.683	1	1.613
GA	70835	2.147	2	2.057
KY	68705	2.082	2	1.995
MD	278514	8.440	8	8.088
MA	475327	14.404	14	13.803
NH	141822	4.298	4	4.118
NJ	179570	5.442	5	5.214
NY	331589	10.048	10	9.629
NC	353523	10.713	10	10.266
PA	432879	13.118	13	12.570
RI	68446	2.074	2	1.988
SC	206236	6.250	6	5.989
VT	85533	2.592	2	2.484
VA	630560	19.108	19	18.310
US	3615920		105	105

No Quota Rule Violation

Problem

	Census 1790	Senate Bill		
State	**Population**	**Divisor 33000**	**Seats**	**Quota *h*=105**
CT	236841	7.177	7	6.877
DE	**55540**	**1.683**	**1**	**1.613**
GA	70835	2.147	2	2.057
KY	68705	2.082	2	1.995
MD	278514	8.440	8	8.088
MA	475327	14.404	14	13.803
NH	141822	4.298	4	4.118
NJ	179570	5.442	5	5.214
NY	331589	10.048	10	9.629
NC	353523	10.713	10	10.266
PA	432879	13.118	13	12.570
RI	68446	2.074	2	1.988
SC	206236	6.250	6	5.989
VT	85533	2.592	2	2.484
VA	**630560**	**19.108**	**19**	**18.310**
US	3615920		105	105

Large states are favored over small states.

Hamilton's Method

State	Population	
CT	236841	
DE	55540	
GA	70835	
KY	68705	
MD	278514	
MA	475327	
NH	141822	
NJ	179570	
NY	331589	
NC	353523	
PA	432879	
RI	68446	
SC	206236	
VT	85533	
VA	630560	
US	3615920	**120.5307**

***d* = 30000**

Hamilton's Method

State	Population	*h* = 120
CT	236841	
DE	55540	
GA	70835	
KY	68705	
MD	278514	
MA	475327	
NH	141822	
NJ	179570	
NY	331589	
NC	353523	
PA	432879	
RI	68446	
SC	206236	
VT	85533	
VA	630560	
US	3615920	**120.5307**

***d* = 30000**

Hamilton's Method

State	Population	*h* = 120	Quota
CT	236841		7.860
DE	55540		1.843
GA	70835		2.351
KY	68705		2.280
MD	278514		9.243
MA	475327		15.774
NH	141822		4.707
NJ	179570		5.959
NY	331589		11.004
NC	353523		11.732
PA	432879		14.366
RI	68446		2.271
SC	206236		6.844
VT	85533		2.839
VA	630560		20.926
US	3615920	**120.5307**	120

$$= 120 \times \frac{236841}{3615920}$$

***d* = 30000**

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q
CT	236841		7.860	7
DE	55540		1.843	1
GA	70835		2.351	2
KY	68705		2.280	2
MD	278514		9.243	9
MA	475327		15.774	15
NH	141822		4.707	4
NJ	179570		5.959	5
NY	331589		11.004	11
NC	353523		11.732	11
PA	432879		14.366	14
RI	68446		2.271	2
SC	206236		6.844	6
VT	85533		2.839	2
VA	630560		20.926	20
US	3615920	**120.5307**	120	111

***d* = 30000**

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q	Appt
CT	236841		7.860	7	8
DE	55540		1.843	1	2
GA	70835		2.351	2	2
KY	68705		2.280	2	2
MD	278514		9.243	9	9
MA	475327		15.774	15	16
NH	141822		4.707	4	5
NJ	179570		5.959	5	6
NY	331589		11.004	11	11
NC	353523		11.732	11	12
PA	432879		14.366	14	14
RI	68446		2.271	2	2
SC	206236		6.844	6	7
VT	85533		2.839	2	3
VA	630560		20.926	20	21
US	3615920	**120.5307**	120	111	120

***d* = 30000**

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q	Appt
CT	236841		7.860	7	8
DE	55540		1.843	1	2
GA	70835		2.351	2	2
KY	68705		2.280	2	2
MD	278514		9.243	9	9
MA	475327		15.774	15	16
NH	141822		4.707	4	5
NJ	179570		5.959	5	6
NY	331589		11.004	11	11
NC	353523		11.732	11	12
PA	432879		14.366	14	14
RI	68446		2.271	2	2
SC	206236		6.844	6	7
VT	85533		2.839	2	3
VA	630560		20.926	20	21
US	3615920	120.5307	120	111	120

This became the first apportionment bill passed by Congress.

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q	Appt
CT	236841		7.860	7	8
DE	55540		1.843	1	2
GA	70835		2.351	2	2
KY	68705		2.280	2	2
MD	278514		9.243	9	9
MA	475327		15.774	15	16
NH	141822		4.707	4	5
NJ	179570		5.959	5	6
NY	331589		11.004	11	11
NC	353523		11.732	11	12
PA	432879		14.366	14	14
RI	68446		2.271	2	2
SC	206236		6.844	6	7
VT	85533		2.839	2	3
VA	630560		20.926	20	21
US	3615920	120.5307	120	111	120

This became the first apportionment bill passed by Congress.

The bill is sent to President Washington for approval.

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q	Appt
CT	236841		7.860	7	8
DE	55540		1.843	1	2
GA	70835		2.351	2	2
KY	68705		2.280	2	2
MD	278514		9.243	9	9
MA	475327		15.774	15	16
NH	141822		4.707	4	5
NJ	179570		5.959	5	6
NY	331589		11.004	11	11
NC	353523		11.732	11	12
PA	432879		14.366	14	14
RI	68446		2.271	2	2
SC	206236		6.844	6	7
VT	85533		2.839	2	3
VA	630560		20.926	20	21
US	3615920	120.5307	120	111	120

This became the first apportionment bill passed by Congress.

The bill is sent to President Washington for approval.

Washington vetoes the bill.

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q	Appt
CT	236841		7.860	7	8
DE	55540		1.843	1	2
GA	70835		2.351	2	2
KY	68705		2.280	2	2
MD	278514		9.243	9	9
MA	475327		15.774	15	16
NH	141822		4.707	4	5
NJ	179570		5.959	5	6
NY	331589		11.004	11	11
NC	353523		11.732	11	12
PA	432879		14.366	14	14
RI	68446		2.271	2	2
SC	206236		6.844	6	7
VT	85533		2.839	2	3
VA	630560		20.926	20	21
US	3615920	120.5307	120	111	120

U.S.:
3615920/120 = 30,132.66…

Hamilton's Method

State	Population	*h* = 120	Quota	Lower Q	Appt
CT	236841		7.860	7	8
DE	55540		1.843	1	2
GA	70835		2.351	2	2
KY	68705		2.280	2	2
MD	278514		9.243	9	9
MA	475327		15.774	15	16
NH	141822		4.707	4	5
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NY	331589		11.004	11	11
NC	353523		11.732	11	12
PA	432879		14.366	14	14
RI	68446		2.271	2	2
SC	206236		6.844	6	7
VT	85533		2.839	2	3
VA	630560		20.926	20	21
US	3615920	120.5307	120	111	120

Connecticut:
236841/8 = 29605.13

Delaware:
55540/2 = 27770

U.S.:
3615920/120 = 30,132.66…

Basic Jefferson Method

After Washington's veto on 5 April 1792, Congress quickly passes the original Senate bill. Washington signed the bill on 14 April 1792.

Two Methodologies

- Divisor Methods

- Quota Methods

Two Methodologies

- Divisor Methods
  - Basic
  - Modified
- Quota Methods

Two Methodologies

- Divisor Methods
  - Basic: *h* is the result
  - Modified
- Quota Methods

Two Methodologies

- Divisor Methods
  - Basic: *h* is the result
  - Modified: *h* is the goal
- Quota Methods

Two Methodologies

- Divisor Methods
  - Basic: *h* is the result
  - Modified: *h* is the goal
- Quota Methods

  h is the resource

Basic Jefferson Method

Basic Jefferson Method

1. Decide on a divisor.

Basic Jefferson Method

1. Decide on a divisor.
2. Calculate each state's quotient:

   quotient = population/divisor

Basic Jefferson Method

1. Decide on a divisor.
2. Calculate each state's quotient:

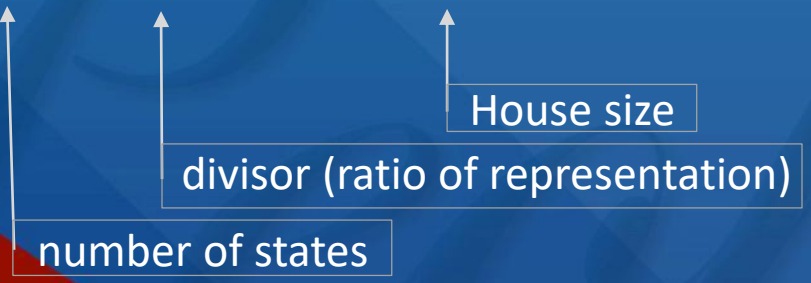
   quotient = population/divisor
3. The state's apportionment is the quotient rounded down.

The resulting house size is the sum of the state apportionments.

First 60 years

A Basic Divisor Method would be used as the House apportionment method until 1850.

- 1790: $s = 15$; $d = 33000 \Rightarrow h = 105$
- 1800: $s = 16$; $d = 33000 \Rightarrow h = 141$
- 1810: $s = 17$; $d = 35000 \Rightarrow h = 181$
- 1820: $s = 24$; $d = 40000 \Rightarrow h = 213$
- 1830: $s = 24$; $d = 47700 \Rightarrow h = 240$
- 1840: $s = 26$; $d = 70680 \Rightarrow h = 223$



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Dean: 46,776 is closer to the target of 50,000; award Vermont 6 seats.

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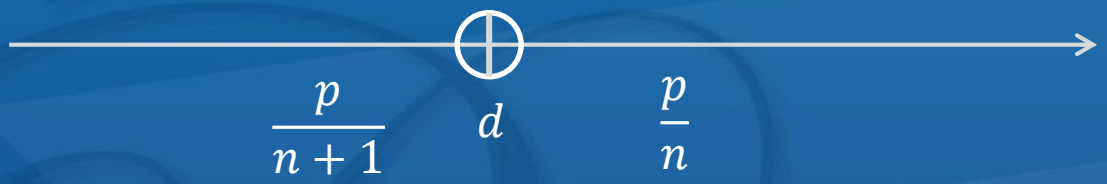

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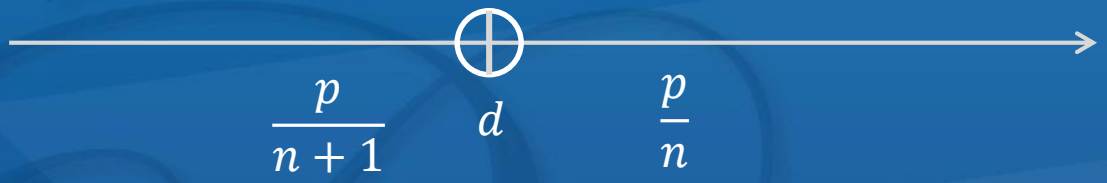

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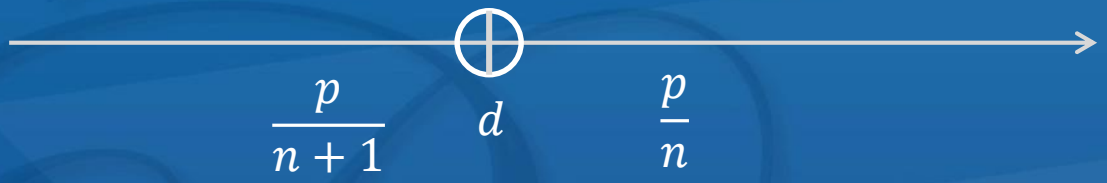

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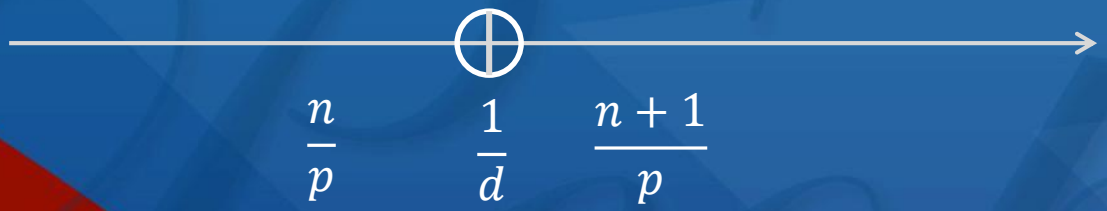

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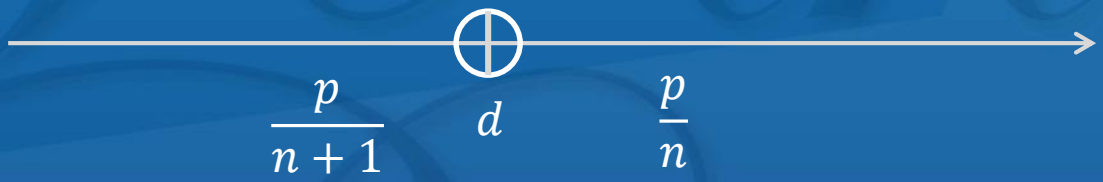

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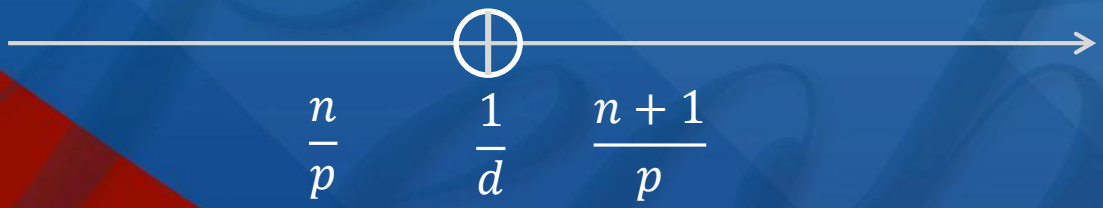

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Adams: round up (max).
Dean: round by closest constituency (HaM).
Webster: round normally (ArM).

In a Round About Way

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GA	210346	6.0099	6	6	6	7
KY	374287	10.6939	10	11	11	11
MD	335946	9.5985	9	10	10	10
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NY	953043	27.2298	27	27	27	28
NC	487971	13.9420	13	14	14	14
OH	230760	6.5931	6	7	7	7
PA	809773	23.1364	23	23	23	24
RI	76931	2.1980	2	2	2	3
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The Apportionment Act of 1842 used a basic divisor method with $d = 70680$ and **Webster's method** of rounding. This yielded $h = 223$, the only time in U.S. history that h decreased as a result of a census-based re-apportionment.

The Vinton Act

The Vinton Act of 1850 (Representative Samuel Vinton, Whig-Ohio) was passed to head off politicizing the census figures. The idea was to adopt a permanent appropriation act.

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But experience exposed problems with the Vinton Act.

Lessons from History

The quota method is subject to counter-intuitive paradoxes:

- The Alabama Paradox
- The Population Growth Paradox

Alabama Paradox

The Alabama paradox may occur when applying the Hamilton quota method:

when the number of House seats is increased, a given state's apportion may decrease.

The Deal Breaker

Results from the 1890 census doomed Hamilton's Method.

House Size	Seats
350 – 382	3
383 – 385	4
386	3
387 – 388	4
389 – 390	3
391 - 400	4

1910

Apportionment based on the 1910 census came from another mutation in apportionment methodology.

Congress abandoned the Quota Method and used a modified divisor method.

Modified Divisor Methods

Step 1. Select the House size, h.

Step 2. Apply a Basic Divisor Method to obtain h seats.

1910: $h = 433$ and Webster's method.

1920 Census

In the 1920 decade, the only time in U. S. History no census-based re-apportionment act was passed.

Congress could not agree on either the House size or the method of apportionment.

The politics of prohibition played a significant role: the dries would not support any proposal that gave the wets more power.

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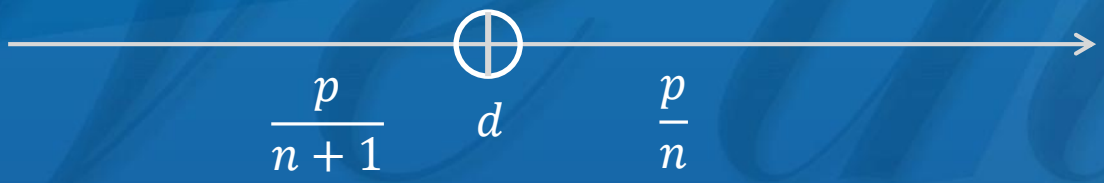

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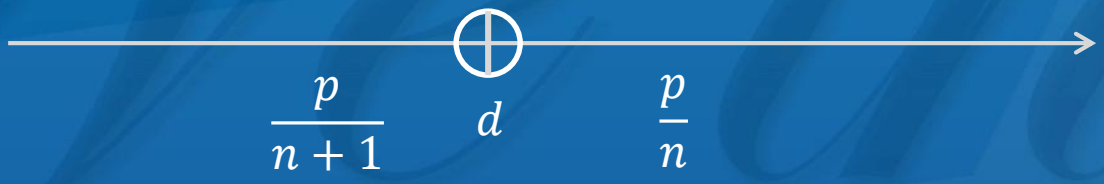

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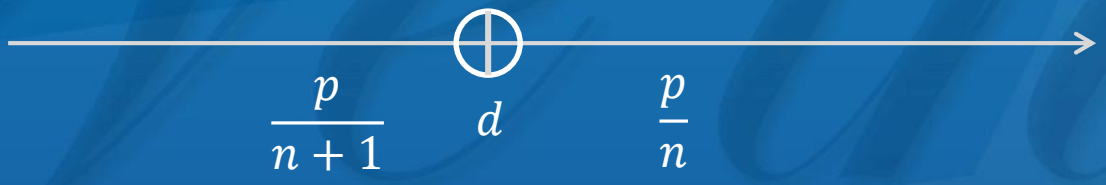


H-H:

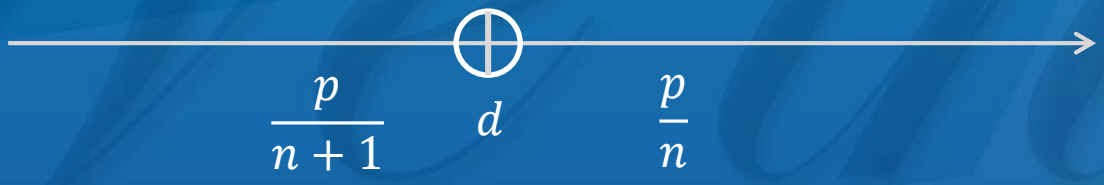
Criterion: $a = n+1$ when $\frac{d}{\left(\frac{p}{n+1}\right)} \leq \frac{\left(\frac{p}{n}\right)}{d}$

Huntington-Hill

Let $q = p/d$ and $n = \text{int}(q)$.

Then $a = n+1$ when $q \geq \text{GeM}(n,n+1)$.

Dean:



H-H:

Criterion: $a = n+1$ when $\frac{d}{\left(\frac{p}{n+1}\right)} \leq \frac{\left(\frac{p}{n}\right)}{d}$

when $\text{GeM}(n,n+1) \leq q$.

The Aftermath

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There are no perfect apportionment methods.

Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

They Mean Well

A modified ***divisor*** method first fixes the House size, then seeks a divisor that when the state's quotients are rounded appropriately and summed, the house size is achieved.

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Distributing 435 seats among 50 states according to their respective populations is a math problem.

Today

Today the Census Bureau obtains apportionments using a priority technique of calculation rather than an ad-hoc technique of calculation.

Census Bureau video 2:09 minutes.
The Amazing Apportionment Machine

Apportionment by Priority

Step 1. Give one seat to each state.

Step 2. Attach a priority number to each state.

Step 3. Award seats one at a time by priority until the desired House size is reached.

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Priority number for a state with n seats

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$$\textit{Priority number for a state with } n \textit{ seats} = \frac{state\ population}{ave(n,n+1)}$$

Five Averages

	ave
• Greatest Divisors	max
• Harmonic Means	HaM
• Equal Proportions	GeM
• Major Fractions	ArM
• Smallest Divisors	min

The Last Seat

Who gots the 435th seat?

	ave	435
• Greatest Divisors	max	IL
• Harmonic Means	HaM	MN
• Equal Proportions	GeM	MN
• Major Fractions	ArM	NC
• Smallest Divisors	min	WA

The Last Seat

Who would get the 436th seat?

	ave	435	436
• Greatest Divisors	max	IL	WA
• Harmonic Means	HaM	MN	CA
• Equal Proportions	GeM	MN	NC
• Major Fractions	ArM	NC	MO
• Smallest Divisors	min	WA	PA

The Last Seat

Who would get the 436th seat?

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• Equal Proportions	GeM	MN	NC
• Major Fractions	ArM	NC	MO
• Smallest Divisors	min	WA	PA

Priority list based on the 2010 census using the method of Equal Proportions.

nia977.wixsite.com/drbcap

Imported From Firefo | 2010 Census Apportio | U. S. Electoral College | One Touch Copy - Dr

Charles M. Biles, Ph.D.

Congressional Apportionment

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Constitutional Congregressional Apportionment Problem

How many seats in the U. S. House of Representatives does each state get?

An answer is presented as an historical narrative with relevant and timely applications in an upcoming book, *The History of Congressional Apportionment.*

Chapter 1. Congressional Apportionment Based on the Census: 1790.
Chapter 2. Congressional Apportionment Based on the Census: 1800-1840.
Chapter 3. Congressional Apportionment Based on the Census: 1850-1890.
Chapter 4. Congressional Apportionment Based on the Census: 1900-1930.
Chapter 5. Congressional Apportionment Based on the Census: 1940-2010.
Chapter 6. An Historical Overture.

NEW

UNDER CONSTRUCTION

Charles Biles

The History of Congressional Apportionment

HSU Press

Last update: 13 June 2017.

Cover Graphic courtesy of The West Virginia Record

Thank You

It is time that I took my seat in this House!

http://www.nia977.wix.com/drbcap

Bonus Resources

Montana

In the 1990 apportionment Montana lost one of its two seats it held for 80 years. In 1991 MT filed suit in federal district court (MT vs. US Dept Commerce).

MT argued the H-H method is unconstitutional and that either Dean's or Adams's method should be used. The federal judges voted 2-1 in favor of MT.

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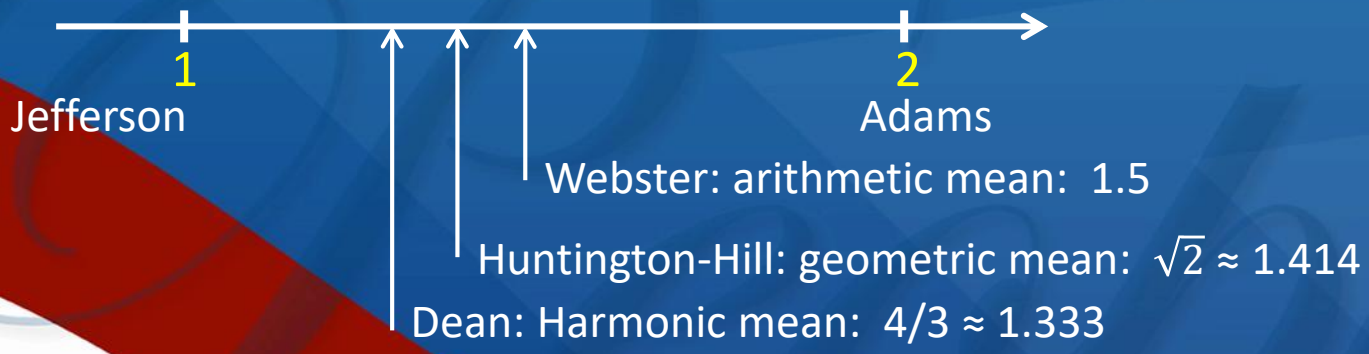

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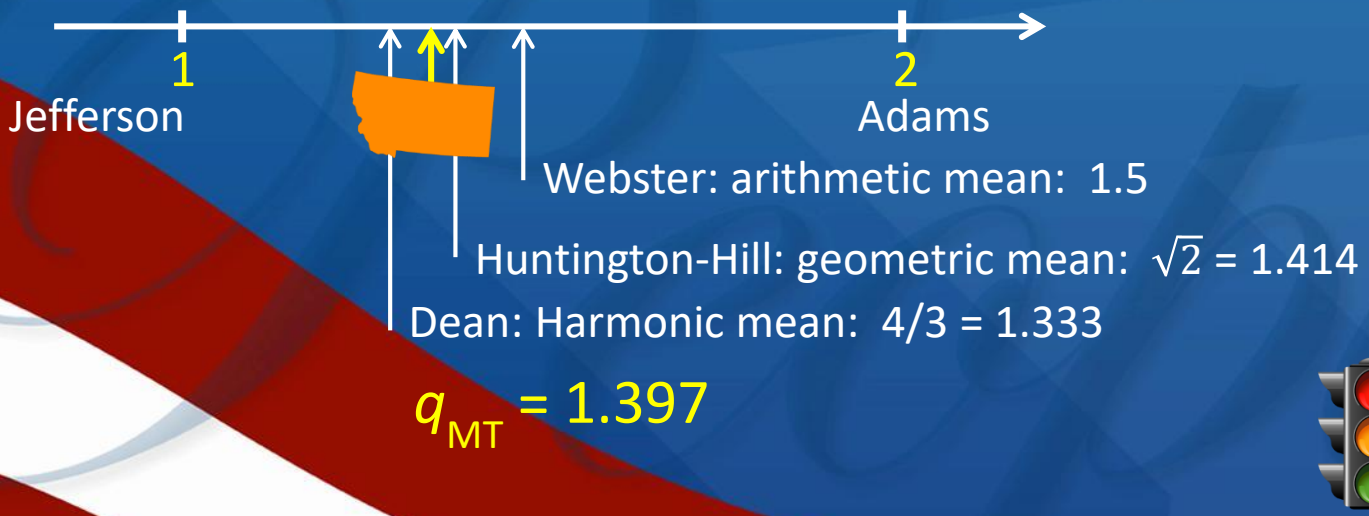

Related Problems

Other problems related to apportionment include:

One Voter, One Vote: The Apportionment of Congressional Seats Reconsidered
Author(s): Howard A. Scarrow
Source: Polity, Vol. 22, No. 2 (Winter, 1989), pp. 253-268
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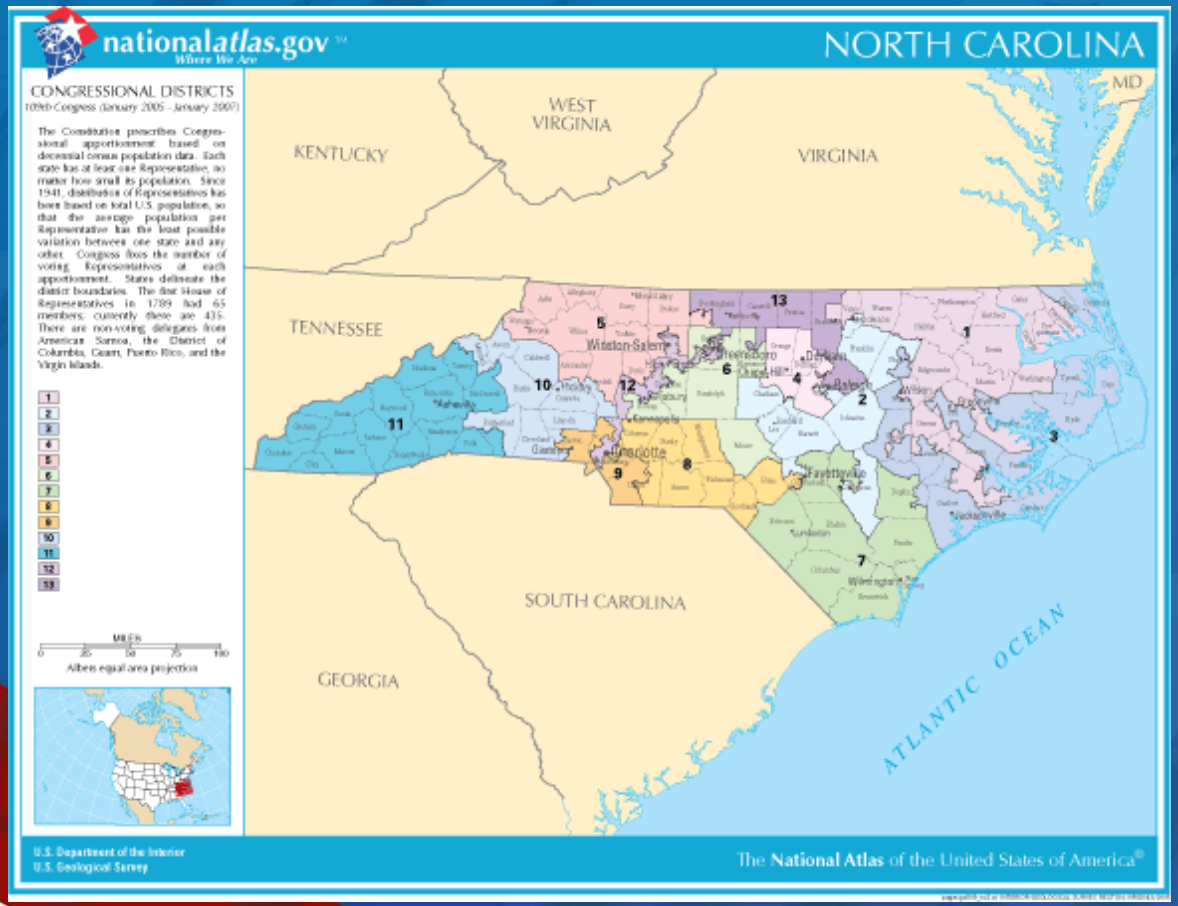
Gerrymandering



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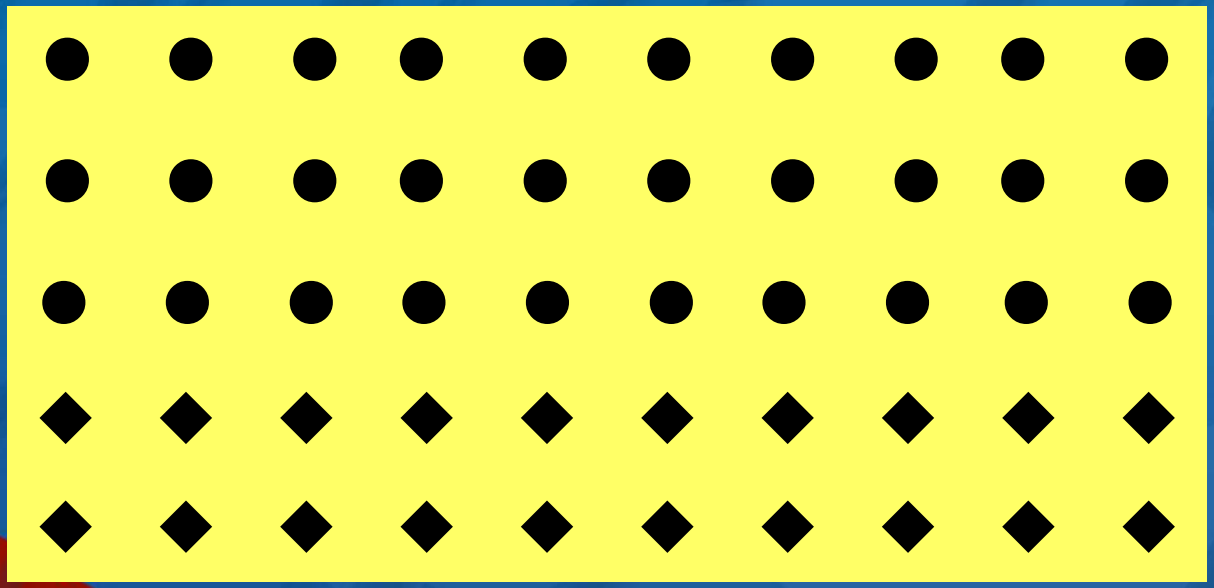
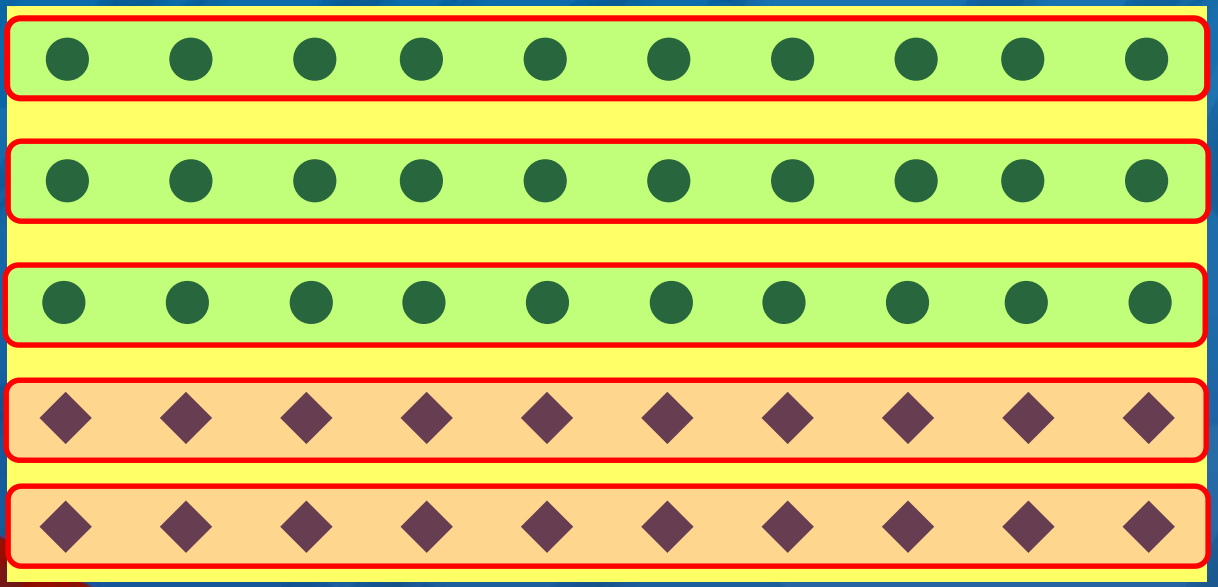


Apportion into five districts.

Gerrymandering



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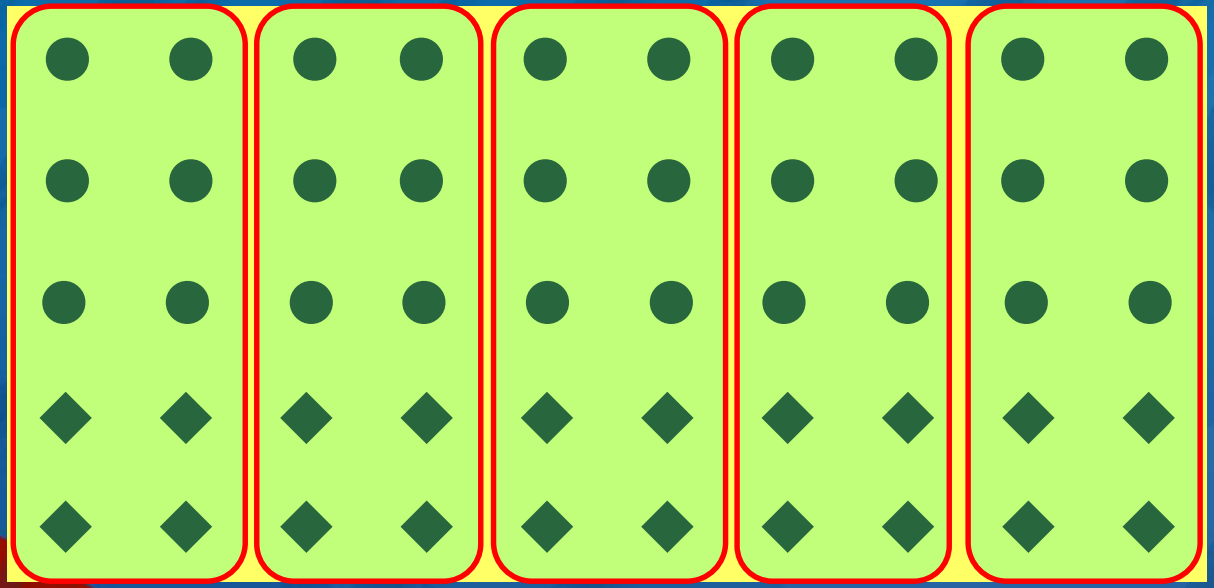

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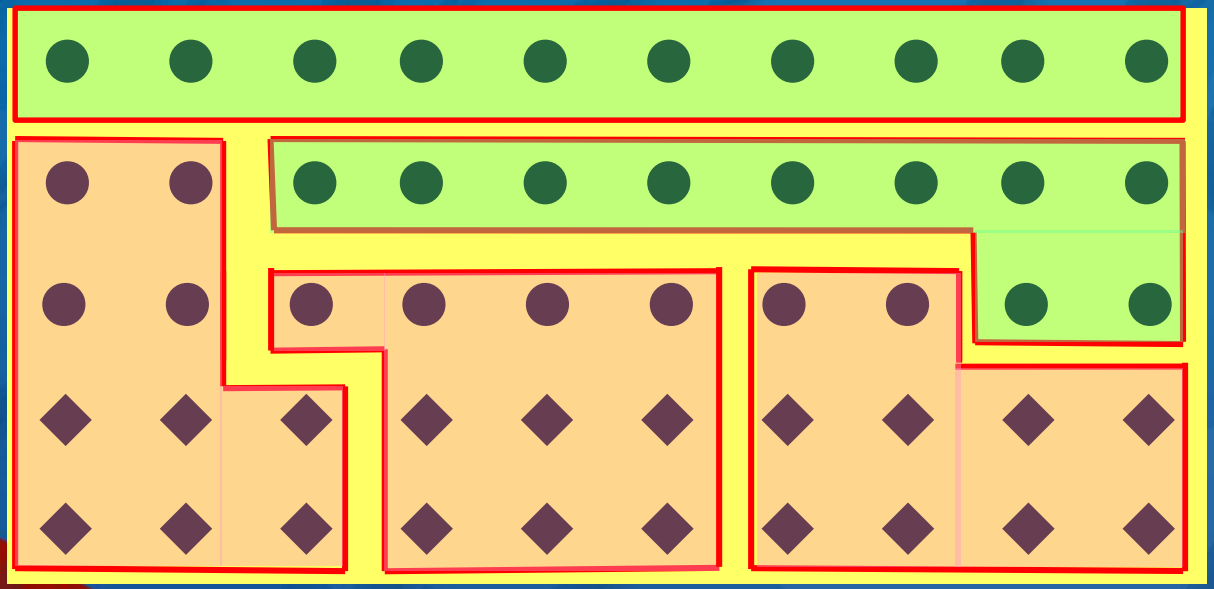

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Washington's Veto

United States [Philadelphia] April 5 1792.

Gentlemen of the House of Representatives

I have maturely considered the Act passed by the two Houses, intitled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.

First—The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.

Second—The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the seperate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.

George Washington.

Copy, DNA: RG 233, Second Congress, 1791–1793, Records of Legislative Proceedings, Journals; LB, DLC:GW. (from Philander Chase, et al., eds.,*The Papers of George Washington, Presidential Series, Vol. 10: March–August 1792* [Charlottesville, Va., 2002], 213-14).

First Apportionment Act

CHAP. XXIII.—*An Act for apportioning Representatives among the several States, according to the first enumeration.*

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachussetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

APPROVED, April 14, 1792.

1790: Why 33000?

State	Population	*d* =	30000	31000	32000	33000	34000	35000	36000	37000	38000	39000	40000
CN	236841		0.8947	0.6400	0.4013	0.1770	0.9659	0.7669	0.5789	0.4011	0.2327	0.0728	0.9210
DE	55540		0.8513	0.7916	0.7356	0.6830	0.6335	0.5869	0.5428	0.5011	0.4616	0.4241	0.3885
GA	70835		0.3612	0.2850	0.2136	0.1465	0.0834	0.0239	0.9676	0.9145	0.8641	0.8163	0.7709
KY	68705		0.2902	0.2163	0.1470	0.0820	0.0207	0.9630	0.9085	0.8569	0.8080	0.7617	0.7176
MD	278514		0.2838	0.9843	0.7036	0.4398	0.1916	0.9575	0.7365	0.5274	0.3293	0.1414	0.9629
MA	475327		0.8442	0.3331	0.8540	0.4038	0.9802	0.5808	0.2035	0.8467	0.5086	0.1879	0.8832
NH	141822		0.7274	0.5749	0.4319	0.2976	0.1712	0.0521	0.9395	0.8330	0.7322	0.6365	0.5456
NJ	179570		0.9857	0.7926	0.6116	0.4415	0.2815	0.1306	0.9881	0.8532	0.7255	0.6044	0.4893
NY	331589		0.0530	0.6964	0.3622	0.0482	0.7526	0.4740	0.2108	0.9619	0.7260	0.5023	0.2897
NC	353523		0.7841	0.4040	0.0476	0.7128	0.3977	0.1007	0.8201	0.5547	0.3032	0.0647	0.8381
PA	432879		0.4293	0.9638	0.5275	0.1175	0.7317	0.3680	0.0244	0.6994	0.3916	0.0995	0.8220
RI	68446		0.2815	0.2079	0.1389	0.0741	0.0131	0.9556	0.9013	0.8499	0.8012	0.7550	0.7112
SC	206236		0.8745	0.6528	0.4449	0.2496	0.0658	0.8925	0.7288	0.5739	0.4273	0.2881	0.1559
VT	85533		0.8511	0.7591	0.6729	0.5919	0.5157	0.4438	0.3759	0.3117	0.2509	0.1932	0.1383
VA	630560		0.0187	0.3406	0.7050	0.1079	0.5459	0.0160	0.5156	0.0422	0.5937	0.1682	0.7640
US	3615920		**8.5307**	**8.6426**	**6.9975**	**4.5733**	**6.3506**	**7.3120**	**9.4422**	**9.7276**	**8.1558**	**5.7159**	**9.3980**
		Unrepresented:	**255920**	**267920**	**223920**	**150920**	**215920**	**255920**	**339920**	**359920**	**309920**	**222920**	**375920**

Alabama Paradox

How is this possible?

State	House 299	House 300
AL	7.646	7.671
TX	9.640	9.672
IL	18.640	18.702

With the House size at 299, Alabama was the last state to be allotted an extra representative to make the House size because of it's decimal. When the House size was increased to 300, all states' quotas were increased by 0.33%. And there were two states that got the extra representatives; and, this time, Texas and Illinois beat out Alabama.

US Census Bureau

The U.S. Census Bureau is housed within the Department of Commerce.

Check out the U.S. Census Bureau for what it says about apportionment.

http://www.census.gov/

Summary 7-page brochure:
http://www.census.gov/prod/cen2010/briefs/c2010br-08.pdf

History of Legislation:
http://www.census.gov/history/www/reference/apportionment/apportionment_legislation_1790_-_1830.html

More!

For playing around, learning or teaching:

http://www.cut-the-knot.org/ctk/Democracy.shtml

Key Decades

The key decades in the history of the Congressional apportionment problem are 1790, 1840 and 1850, and 1920. Here are some excellent resources for each of these periods.

- Edmund J. James, *The First Apportionment of Federal Representatives in the United States*, Annals of the American Academy of Political and Social Science, 9 (January 1897): 1-41.
- Johanna Nicol Shields, *Whigs Reform the "Bear Garden": Representation and the Apportionment Act of 1842*, Journal of the Early Republic, 5 (Fall 1983): 356-82.
- Charles W. Eagles, *Democracy Delayed: Congressional Reapportionment and Urban-Rural Conflict in the 1920s*, University of Georgia Press, 1990.

US History

For any serious research of U.S. history, one must know about the Journals of Congress which includes the House Journal and the Senate Journal:

http://memory.loc.gov/ammem/amlaw/lwhj.html

The Future: Reform?

Four Proposals:

The Future: Reform?

Four Proposals:

- Thirty-thousand.org
- Wyoming Rule
- Neubauer and Gartner
- Webster's Method.

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

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Comment: This leads to a House with 10283 (10306) representatives.

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Jefferson basic divisor method.

CA: 1244 seats!

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

Thirty-thousand.org advocates 50000/representative.

This creates a House with 6181 representatives using Webster's method of rounding.

California gets 747 seats.

The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).

http://en.wikipedia.org/wiki/Wyoming_Rule

http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/

The Wyoming Rule

Results of applying the WY Rule to the 2000 and 2010 censuses.

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2000 smallest state: WY, 493782.

$h = 569$ Huntington-Hill

The Wyoming Rule

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2000 smallest state: WY, 493782.

h = 569 Huntington-Hill

2010 smallest state: WY, 563626

h = 543 Dean HI

h = 542 Huntington-Hill

h = 540 Webster NJ, SD

A Proposal

A Proposal for Apportioning the House

Michael G. Neubauer, CSU Northridge, Mathematics
Margo G. (Gartner) Carr, Cerro Coso Community College

…the problem of finding a “good” house size and “right” apportionment method are best considered together.

Source: PSC 44(1), January 2011: 1—3.

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Definition. A House size is *agreeable* means that the apportionments by the methods of Hamilton, Dean, Huntington-Hill, and Webster all agree.

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Source: PSC 44(1), January 2011: 1—3.

Webster's Method

The simplest reform would be to replace the geometric mean of decimal rounding in the Huntington-Hill method by the arithmetic mean of decimal rounding in Webster's method.

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The research of Balinski and Young has produced the following two key results. Since the Alabama paradox is a deal-breaker, then congressional apportionment must be based on a divisor method. Further, Webster's is the only rounding method that is unbiased towards either larger or smaller states.