Now and the Future

Charles Biles, Ph.D.
Mathematics 5: Contemporary Mathematics
College of the Redwoods
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"… no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, …"

James Madison
The Federalist 55

The Apportionment Problem

Determine how many seats in the U.S. House of Representatives each state gets.

CONGRESSIONAL SEATS

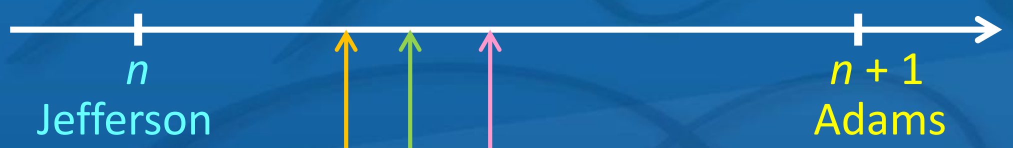
2010 OFFICIAL RESULTS

(US apportionment population = 309,183,463)$/435 \approx 710{,}767$

http://www.census.gov/2010census/data/apportionment-data.php

They Mean Well

A modified ***divisor*** method first fixes the House size, then seeks a divisor that when the state's quotients are rounded and summed, the house size is achieved.



Webster: arithmetic mean $\dfrac{n+(n+1)}{2} = n + \dfrac{1}{2}$

Huntington-Hill: geometric mean $\sqrt{n(n+1)}$

Dean: harmonic mean $\dfrac{2}{\dfrac{1}{n}+\dfrac{1}{n+1}} = \dfrac{2n(n+1)}{2n+1}$

Montana

In the 1990 apportionment, Montana lost one of its two seats it held for 80 years. In 1991 MT filed suit in federal district court (MT vs. US Dept Commerce).

MT argued the H-H method is unconstitutional and that either Dean's or Adams's method should be used. The federal judges voted 2-1 in favor of MT.

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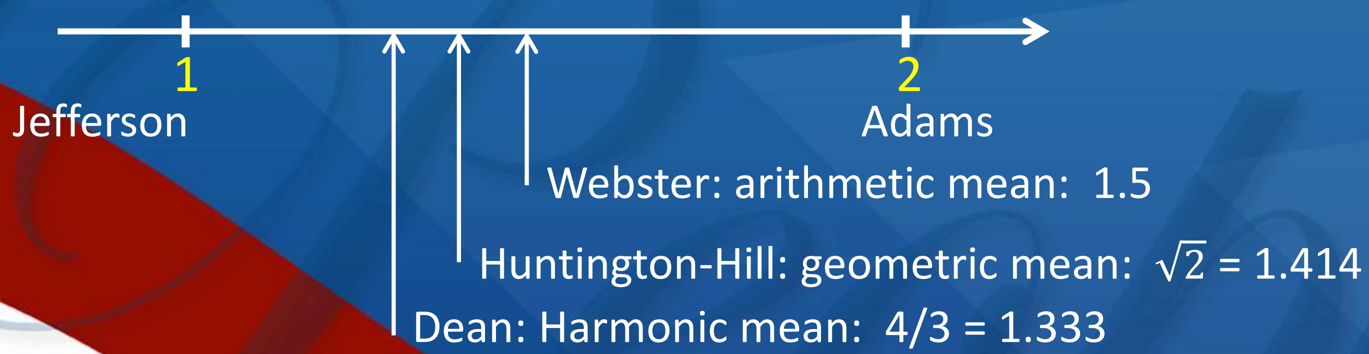

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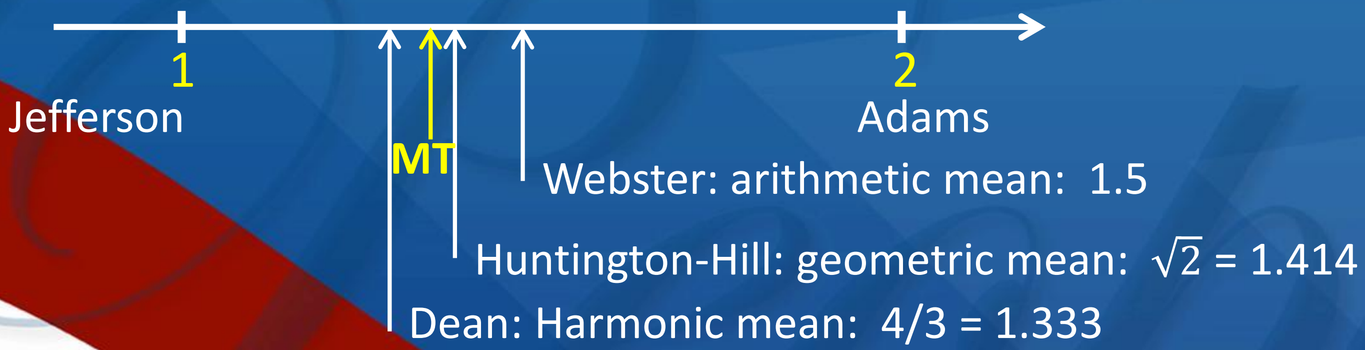

$q_{MT} = 1.397$

Apportionment Problems

On appeal the U.S. Supreme Court unanimously ruled that the H-H method was constitutional. The district court's decision was overturned.

http://www.law.cornell.edu/supct/html/91-860.ZS.html

http://caselaw.lp.findlaw.com/scripts/getcase.pl?court=US&vol=503&invol=442

Today

https://www.census.gov/library/video/census_apportionment_machine.html

Today

Today the Census Bureau obtains apportionments using a priority technique of calculation rather than an ad-hoc technique of calculation.

An Average Lesson

1. How to average two different positive numbers.
2. How to round a positive decimal number.

An Average Lesson

1. The average of *a* and *b* where $0 < a < b$.

$$\text{ave}(a,b) = \max(a,b) = b$$

$$\min(a,b) = a$$

$$\text{AM}(a,b) = (a+b)/2$$

$$\text{GM}(a,b) = \sqrt{a \times b}$$

$$\text{HM}(a,b) = \frac{2}{\left(\frac{1}{a}+\frac{1}{b}\right)} = \frac{2ab}{a+b}$$

Ad-hoc Modified Divisor

Step 1. Decide the House size: *h*.

Step 2. Apply a basic divisor method to obtain the preset *h*.

Serial Distribution

Step 1. Award 1 seat to each state.

This distributes 50 seats.

Step 2. Then award the 51^{st} seat, 52^{nd} seat, 53^{rd} seat, etc., according to a list of priority numbers.

Priority Numbers

2010 Census

Seat	Priority	State	Apportionment
434	711308	CA	53
435	710231	MN	8
436	709063	NC	14
437	708459	MO	9
438	706337	NY	28
439	705164	NJ	13
440	703158	MT	2

http://www.census.gov/data/tables/2010/dec/2010-apportionment-data.html

Priority Numbers

$$\mathrm{PN}(n) = \frac{\textit{population}}{\mathrm{ave}(n,n+1)}$$

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where $\mathrm{ave}(n,n+1)$ =

Jefferson:	$\max(n,n+1)$
Dean:	$\mathrm{HM}(n,n+1)$
Huntington-Hill:	$\mathrm{GM}(n,n+1)$
Webster:	$\mathrm{AM}(n,n+1)$
Adams:	$\min(n,n+1)$

Priority Numbers

$$\mathbf{PN}(n) = \frac{\textit{population}}{\mathrm{ave}(n,n+1)}$$

where ave(n,n+1) =

Jefferson:	max(n,n+1)	Largest Divisors
Dean:	HM(n,n+1)	Harmonic Mean
Huntington-Hill:	GM(n,n+1)	Equal Proportions
Webster:	AM(n,n+1)	Major Fractions
Adams:	min(n,n+1)	Smallest Divisors

Priority Numbers

Census 1790	
State	Population
Connecticut	236841
Delaware	55540
Georgia	70835
Kentucky	68705
Maryland	278514
Massassachutts	475327
New Hampshire	141822
New Jersey	179570
New York	331589
North Carolina	353523
Pennsylvania	432879
Rhode Island	68446
South Carolina	206236
Vermont	85533
Virginia	630560
United States	3615920

Priority Numbers

Census 1790		
State	Population	Seats
Connecticut	236841	1
Delaware	55540	1
Georgia	70835	1
Kentucky	68705	1
Maryland	278514	1
Massassachutts	475327	1
New Hampshire	141822	1
New Jersey	179570	1
New York	331589	1
North Carolina	353523	1
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Virginia	630560	1
United States	3615920	15

Huntington - Hill

$$\mathrm{PN}(1) = p/\sqrt{1 \times 2} = p/\sqrt{2}$$

Priority Numbers

Census 1790			H-H
State	Population	Seats	Priority
Connecticut	236841	1	167471
Delaware	55540	1	39272
Georgia	70835	1	50087
Kentucky	68705	1	48581
Maryland	278514	1	196939
Massassachutts	475327	1	336106
New Hampshire	141822	1	100283
New Jersey	179570	1	126975
New York	331589	1	234468
North Carolina	353523	1	249978
Pennsylvania	432879	1	306091
Rhode Island	68446	1	48398
South Carolina	206236	1	145830
Vermont	85533	1	60480
Virginia	630560	1	445873
United States	3615920	15	

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United States	3615920	15	

Priority Numbers

16 VA 2

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United States	3615920	16	

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Huntington - Hill

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Priority Numbers

16 VA 2
17 MA 2
18 PA 2
19 VA 3

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Virginia	630560	3	**257425**
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Huntington - Hill

$PN(1) = p/\sqrt{1 \times 2} = p/\sqrt{2}$

$PN(2) = p/\sqrt{2 \times 3} = p/\sqrt{6}$

$PN(3) = p/\sqrt{3 \times 4} = p/\sqrt{12}$

Priority Numbers

16	VA	2
17	MA	2
18	PA	2
19	VA	3

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State	Population	Seats	Priority
Connecticut	236841	1	167471
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Huntington - Hill

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$$PN(3) = p/\sqrt{3 \times 4} = p/\sqrt{12}$$

Today

$$A_n = \frac{P}{ave(n, n+1)}$$

$$A_n = \frac{P}{\sqrt{n \times (n+1)}}$$

The Future: Reform?

Four Proposals:

The Future: Reform?

Four Proposals:

- Thirty-thousand.org
- The Wyoming Rule
- Neubauer and Gartner
- Webster's Method

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

thirty-thousand.org

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http://www.thirty-thousand.org/

Comment: This leads to a House with 10283 representatives.

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

Comment: This leads to a House with 10283 representatives.

CA: 1244 seats!

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

Thirty-thousand.org advocates 50000/representative.

This leads to a House with 6181 representatives using Webster's method of rounding.

California gets 747 seats.

The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).

http://en.wikipedia.org/wiki/Wyoming_Rule

http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/

The Wyoming Rule

Here are the results of applying the WY Rule to the 2000 and 2010 censuses.

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2000 smallest state: WY, 493782.

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2000 smallest state: WY, 493782.

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2010 smallest state: WY, 563626

h = 543 Dean HI

h = 542 Huntington-Hill

h = 540 Webster NJ, SD

A Proposal

A Proposal for Apportioning the House

Michael G. Neubauer, CSU Northridge, Mathematics

Margo G. (Gartner) Carr, Fordham University

…the problem of finding a "good" house size and "right" apportionment method are best considered together.

Source: PSC 44(1), January 2011: 1—3.

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Source: PSC 44(1), January 2011: 1—3.

Webster's Method

The simplest reform would be to replace the geometric mean of decimal rounding in the Huntington-Hill method by the arithmetic mean of decimal rounding in Webster's method.

Webster’s Method

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The research of Balinski and Young has produced two key results. Since the Alabama paradox is a deal-breaker, then congressional apportionment must be based on a divisor method.

Further, Webster’s is the only rounding method that produces results that are unbiased towards either larger or smaller states.

An Application

An Application

Resolved: The electoral system for electing the President should be replaced by the popular vote.

An *Election* is an example of a basic problem.

An *Election* is an example of a basic problem.

How can one say something informative about a group when the individuals in the group are all different?

Voters

Ballots

Who wins?

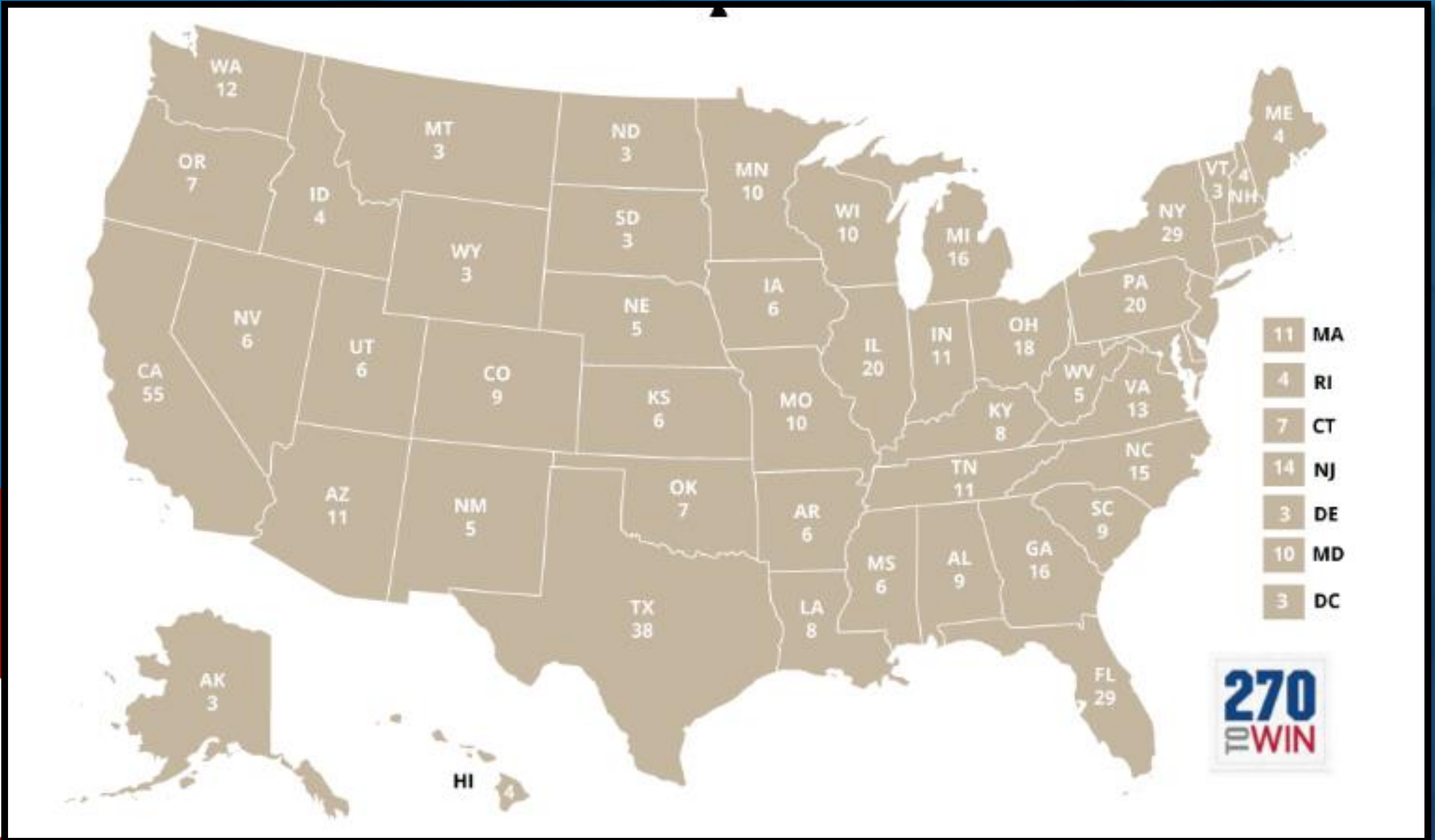
Ballots

What do you see?

Two Models

- Electoral College Model
- States Model

Electoral College Model



N = 538, vote for one

States Model

N = 51, weighted ballots

The States Model

In a presidential election in The United States the *electorate* consists of the 50 states plus the District of Columbia.

The States Model

In a presidential election in The United States the *electorate* consists of the 50 states plus the District of Columbia.

The *ballot* is not one-state, one-vote. The ballot is a weighted ballot as determined by the *electoral system.*

The Electoral College

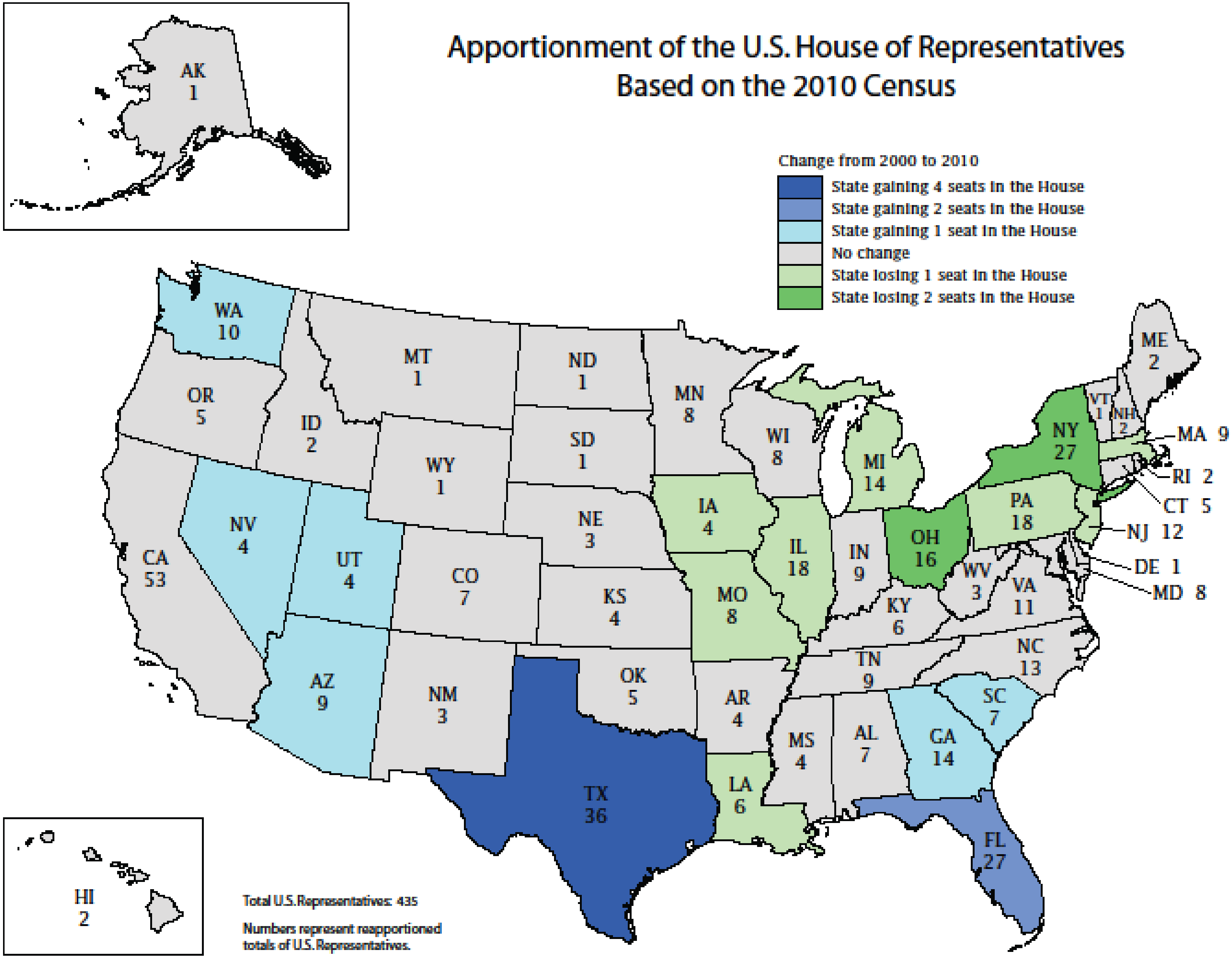
The President is elected by a majority vote of the electors as specified by the U. S. Constitution, Article II, Section 1 with Amendment XII (ratified in 1804) and Amendment XX (ratified in 1933).

The Electoral College

The College consists of a slate of electors from each state. **The number of electors equals the number of members of Congress—the number of representatives in the House plus two senators.**

Amendment XXIII (ratified 1961) allows the District of Columbia a slate of three electors.

Apportionment of the U.S. House of Representatives Based on the 2010 Census



U.S. Department of Commerce

USCENSUSBUREAU

Small State Bias

The Electoral College is heavily weighted towards smaller states.

California has 66 times the population of Wyoming.

The electoral vote ratio is CA 55 and WY 3.

The Aftermath

Michel Balinski, Professor of Mathematics at SUNY Stony Brook and H. Peyton Young, Professor of Mathematics at Johns Hopkins, proved the following theorem in 1982:

There are no perfect apportionment methods.

Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

The Presidential Election

In each state except Maine and Nebraska, the electoral slate is awarded to the plurality winner of the state's popular vote, known as "winner take all."

When you cast a vote for candidate X in a presidential election, you are casting a vote for X's slate of electors in your state.

The Presidential Election

In Maine (1972) and Nebraska (1992), two electoral votes go to the statewide plurality winner. The remaining electoral votes are distributed to the plurality winner of each congressional district.

The Presidential Election

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In 2016 in Maine Clinton won 3 electoral votes and Trump 1 electoral vote.

The 2016 Presidential Election

The popular vote:

Hillary Clinton:	65,844,610	48.2%
Donald Trump:	62,979,636	46.1%
Others:	7,804,213	5.7%

Certified Results:
http://cookpolitical.com/story/10174

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The popular vote:

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The Electoral College vote:

Hillary Clinton:	227
Donald Trump:	304
Others:	7

The Popular Vote

Arguably, on 4 other occasions in U. S. history the electoral and popular systems produced different results.

1. John Quincy Adams vs. Andrew Jackson 1824
2. Rutherford B. Hayes vs. Samuel Tilden 1876
3. Benjamin Harrison vs. Grover Cleveland 1888
4. George Bush vs. Al Gore 2000

1876

Candidate	Party	Popular Vote	Electoral Vote
Rutherford B. Hayes (OH)	Republican	4,034,142	185
Samuel J. Tilden (NY)	Democratic	4,286,808	184
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The original apportionment bill based on the 1870 census used the Hamilton Quota Method. The 1872 supplement bill added nine seats but used a different method. The supplement's method flipped seats for Illinois and New York to New Hampshire and Florida.

2000

Candidate	Party	Popular Vote	Electoral Vote
George W. Bush (TX)	Republican	5,443,633	271
Albert Gore (TN)	Democratic	5,538,163	266
Ralph Nader (DC)	Green	250,017	---
Patrick Buchanan (VA)	Reform	149,115	---

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Lesson: Bush's Electoral College victory was an artifact of the size of the House of Representatives.

Neubauer and Zeitlin

Neubauer and Zeitlin calculated the Electoral College vote based on House sizes 50 - 1000 using the current method of congressional apportionment.

As the House size ranges from 50 to 1000, the 2000 election would have produced Electoral College ties for 25 House sizes.

For all House sizes larger than 597, except 655 which produces a tie, Gore wins. For all House sizes smaller than 491 Bush wins.

Neubauer and Zeitlin

In the intermediate range 492-597 the winner oscillates randomly between Bush and Gore.

For these 106 House sizes, Bush and Gore tie 24 times, Bush wins 53 times, and Gore wins 29 times.

http://www.thirty-thousand.org/pages/Neubauer-Zeitlin.htm

2000

The 2000 election displays another potential problem. The number of electoral votes each state gets is tied to the decennial census. Although the election was in 2000, apportionment of the House was based on the 1990 census. An election held in a census year is based on a population that is ten years old.

What would have been the result of Bush vs. Gore if the Electoral College vote were based on the 2000 census for congressional apportionment?

2000

In comparison with the 1990 census the 2000 census affected congressional apportionment for eighteen states:

Arizona, gain 2; California, gain 1; Colorado, gain 1; Connecticut, lose 1; Florida, gain 2; Georgia, gain 2; Illinois, lose 1; Indiana, lose 1; Michigan, lose 1; Mississippi, lose 1; Nevada, gain 1; New York, lose 2; North Carolina, gain 1; Ohio, lose 1; Oklahoma, lose 1; Pennsylvania, lose 2; Texas, gain 2; Wisconsin, lose 1.

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Accordingly, the electoral vote would have changed from

Bush 271 and Gore 266
to
Bush 277 and Gore 259.

Reform

Over the past 200 years, over 700 proposals have been introduced in Congress to reform or eliminate the Electoral College. There have been more proposals for Constitutional amendments on changing the Electoral College than on any other subject.

https://www.archives.gov/federal-register/electoral-college/faq.html#whyec

Today's Debate

Resolved: The electoral system should be replaced by a popular vote system.

What’s the Popular Vote System?

The 2016 Presidential Election

The popular vote:

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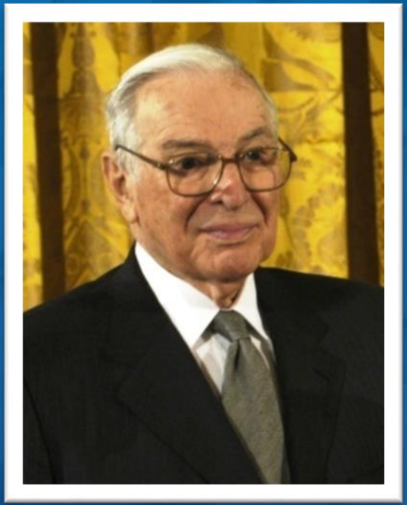

Kenneth Arrow
Nobel Prize in Economics 1972

There is no voting system that can satisfy basic requirements of fairness in all cases.

Fairness Axioms

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- Individual Sovereignty

Question 1

Should there be a uniform national presidential ballot?

- Should there be a national ballot access law?

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Ballots differ from state to state. GA, IN, OK only listed 3 candidates; CA 5; TN 7; UT 10; CO 22.

http://www.politico.com/2016-election/results/map/president

Question 1

Should there be a uniform national presidential ballot?

- Should there be a national ballot access law?
- Should there be a write-in provision?
  Today nine states do not allow a write-in.

https://ballotpedia.org/Ballot_access_for_presidential_candidates

Question 2

Should there be a uniform national standard for voter suffrage?

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- Should all American citizens "in good-standing" be allowed to vote in the national popular election for President?

http://felonvoting.procon.org/view.resource.php?resourceID=000286

Question 2

Should there be a uniform national standard for voter suffrage?

- Should all American citizens "in good-standing" be allowed to vote in the national popular election for President?
- What about American citizens who live in a U.S. territory but are not citizens of a state or residents of D. C.?

Question 3

What should be the structure of the ballot in a national presidential election?

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What should be the structure of the ballot in a national presidential election?

- Vote for one.
- Approval voting.
- Ranked choice voting.

The Ballot

An election must feature a ballot. We will assume the ballot is the same for each voter; further, one person/one ballot.

The Ballot

The structure of the ballot determines your voice in an election.

The Ballot

A two-option ballot looks like this:

Vote for One
❑ Option A
❑ Option B

The Single Vote Ballot

A multi-option ballot looks like this:

Instruction

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

The Single Vote Ballot

A multi-option ballot looks like this:

Vote for One

- ❑ Option A
- ❑ Option B
- ❑ Option C
- ❑ Option D
- ❑ Option E

Approval Voting

A multi-option ballot looks like this:

Approval List

- ❑ Option A
- ❑ Option B
- ❑ Option C
- ❑ Option D
- ❑ Option E

Vote for all options that you approve.

Ranked Choice Voting

In a ranked choice ballot the voter ranks some or all of the candidates.

In a top three system, you rank your top 3 choices as 1, 2, or 3.

In a full ranked system, if the ballot displays five choices, then you rank those choices 1 though 5.

News Flash In the 2016 elections Maine approved a full ranked system for statewide offices.
https://ballotpedia.org/Maine_Ranked_Choice_Voting_Initiative,_Question_5_(2016)

Instant Runoff Voting

Rank the options

Option	Rank 1	2	3	4	5
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○
E	○	○	○	○	○

Instant Runoff Voting

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the options

Option	Rank 1	2	3	4	5
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○
E	○	○	○	○	○

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	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

To win: majority of first place votes.

Here, no candidate gets a majority of first place votes.

Instant Runoff Voting

	18	12	10	9	4	2
First	A	B	C	D	E	E
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Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Eliminate the “least fit” candidate and then recount the votes.

Instant Runoff Voting

	18	12	10	9	4	2
First	A	C	C	C	C	C
Second	C	A	A	A	A	A
Third						
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.

Next eliminate B.

C wins: 37-18 !

Instant Runoff Voting

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Question: How many votes did C get?

State Sovereignty

Or, we could just keep the election as a "state's rights" matter.

Ballot access, ballot structure, suffrage, voting mechanics would be left up to each state. Then count the current popular vote in each state as is currently done.

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What could possibly go wrong?

Should the electoral system be replaced by a popular vote system?

What is an election?

Thank You

It is time that I took my seat in this House!

http://www.nia977.wix.com/drbcap

Bonus Resources

Related Problems

Other problems related to apportionment include:

One Voter, One Vote: The Apportionment of Congressional Seats Reconsidered
Author(s): Howard A. Scarrow
Source: Polity, Vol. 22, No. 2 (Winter, 1989), pp. 253-268
Published by: Palgrave Macmillan Journals
Stable URL: http://www.jstor.org/stable/3234834 .

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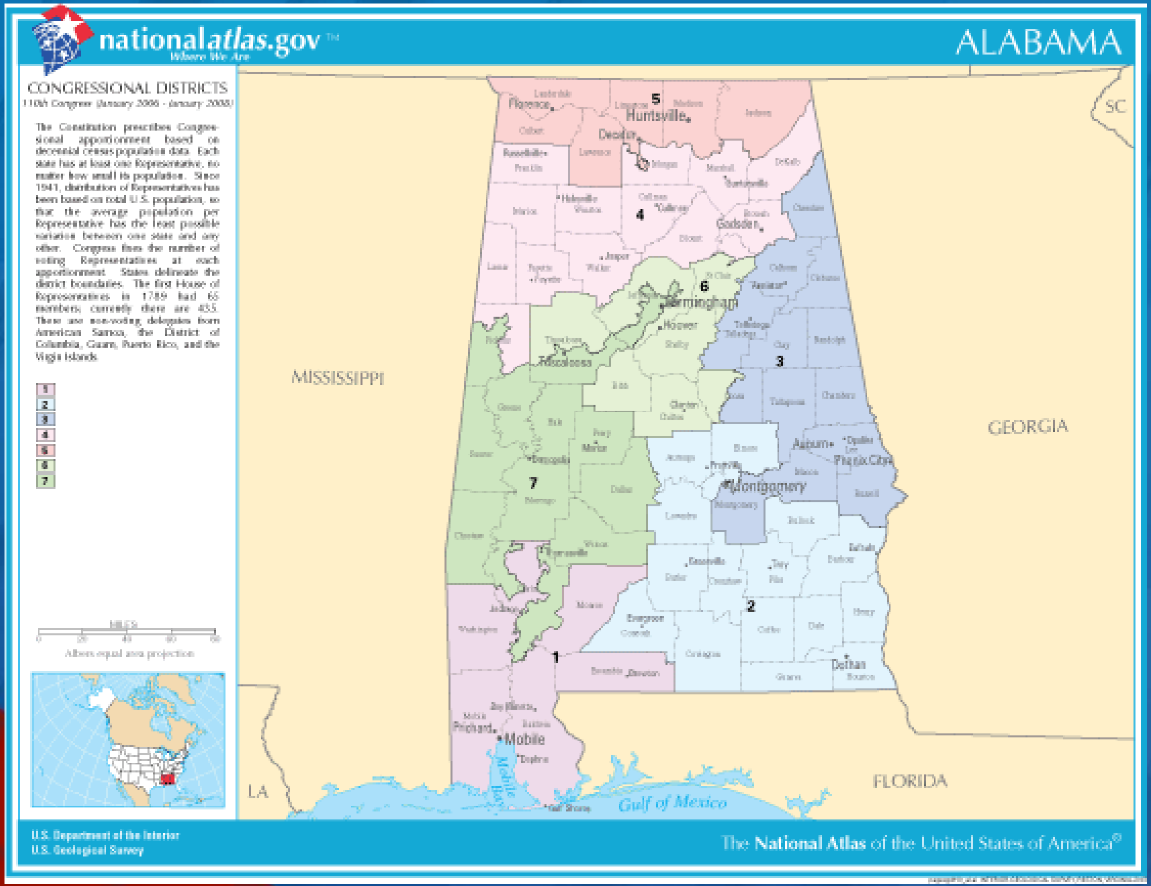
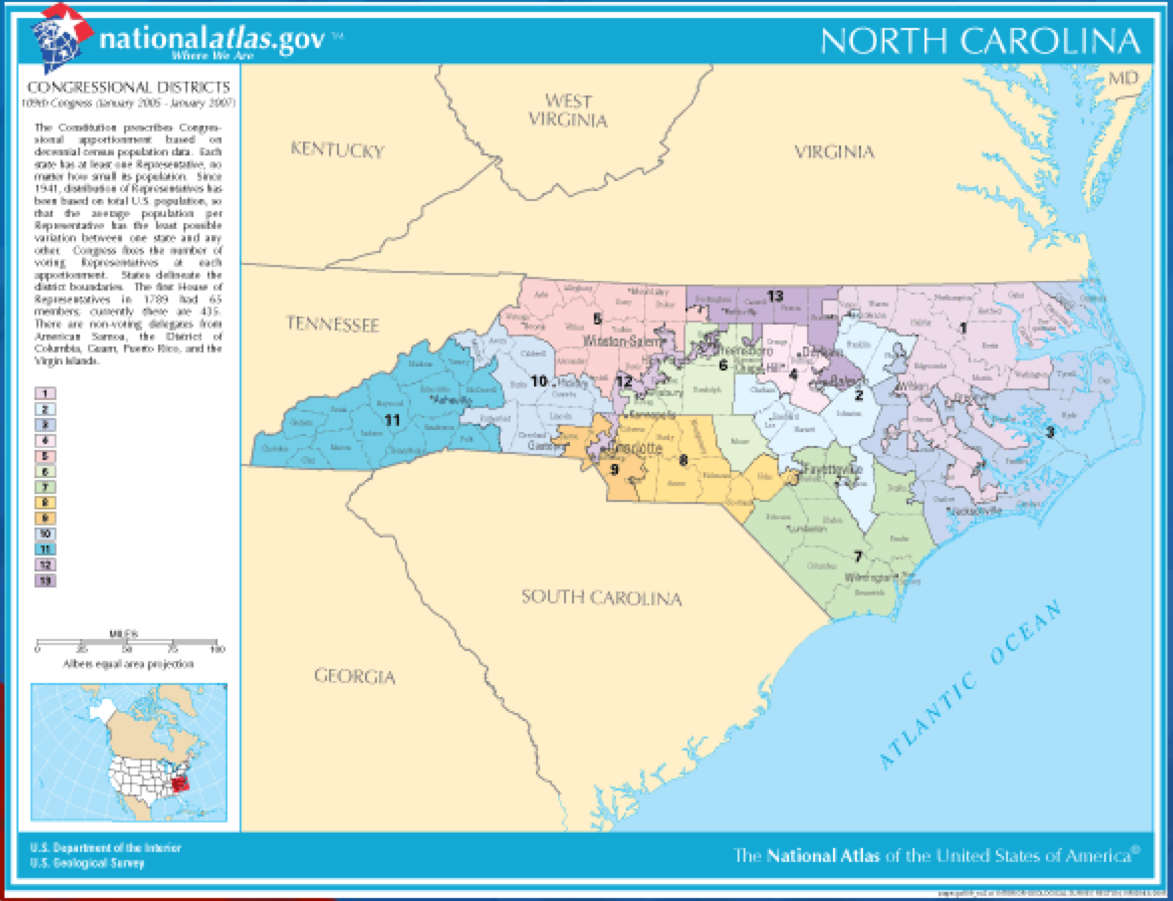
Gerrymandering



http://www.nationalatlas.gov/printable/congress.html#al

Gerrymandering



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- The Ballot Options
- Voting: the mechanism of voting.
- Decision: how does one decide the winner?

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The Apportionment Problem

The Problem is nicely explained in the website:

http://www.ams.org/samplings/feature-column/fcarc-apportion1

Presidential Elections

The 1870s saw a new twist in apportionment that spilled over into a Presidential election. In the apportionment of 1871, the House size was set to 292. Hamilton’s method was legally in place. Yet the actual apportionment approved by Congress differed in four states from the Hamilton apportionment. NY was assigned 33 seats, IL 19, NH 3, and FL 2. But Hamilton’s method would have given NY 34, IL 20, NH 2, and FL 1. Whatever Congress may have intended, the apportionment they approved is one that would have been given by Dean’s method for the Census of 1870.

Source:
http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=3163&pf=1

Presidential Elections

Why is this such a big deal? In the closely contested election of 1876, Samuel Tilden won NY while his opponent, Rutherford B. Hayes, won the other three states. Hayes beat Tilden in the Electoral College 185 to 184. Had Hamilton's method been followed, the count in the College would have been reversed and Tilden would have been elected!

See the spreadsheet 1876 apportion for an illustration of the Hamilton calculation as compared to the actual apportionment and for a tabulation of the electoral votes in the election of 1876.

Presidential Elections

So in 1876, Hayes won under a Dean apportionment but would have lost under a Hamilton apportionment, even if no other factors had changed. Now let's jump forward to the Presidential election of 2000. In the Electoral College, George W. Bush defeated Al Gore by a tally of 271 to 266. (Gore should have had 267 votes, but one of his electors from Washington, D.C. abstained.) Had the Congress used Jefferson's method to apportion the House after the 1990 census, Gore would have garnered 271 electoral votes and become the President. Even more intriguingly, had Hamilton's method been in place, the Electoral College vote would have been tied at 269 and the election thrown to the House of Representatives for resolution. Methods of apportionment do have practical consequences!

Washington’s Veto

United States [Philadelphia] April 5 1792.

Gentlemen of the House of Representatives

I have maturely considered the Act passed by the two Houses, intitled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.

First—The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.

Second—The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the seperate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.

George Washington.

Copy, DNA: RG 233, Second Congress, 1791–1793, Records of Legislative Proceedings, Journals; LB, DLC:GW. (from Philander Chase, et al., eds.,*The Papers of George Washington, Presidential Series, Vol. 10: March–August 1792* [Charlottesville, Va., 2002], 213-14).

First Apportionment Act

CHAP. XXIII.—*An Act for apportioning Representatives among the several States, according to the first enumeration.*

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachussetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

APPROVED, April 14, 1792.

1790: Why 33000?

State	Population	*d* =	30000	31000	32000	33000	34000	35000	36000	37000	38000	39000	40000
CN	236841		0.8947	0.6400	0.4013	0.1770	0.9659	0.7669	0.5789	0.4011	0.2327	0.0728	0.9210
DE	55540		0.8513	0.7916	0.7356	0.6830	0.6335	0.5869	0.5428	0.5011	0.4616	0.4241	0.3885
GA	70835		0.3612	0.2850	0.2136	0.1465	0.0834	0.0239	0.9676	0.9145	0.8641	0.8163	0.7709
KY	68705		0.2902	0.2163	0.1470	0.0820	0.0207	0.9630	0.9085	0.8569	0.8080	0.7617	0.7176
MD	278514		0.2838	0.9843	0.7036	0.4398	0.1916	0.9575	0.7365	0.5274	0.3293	0.1414	0.9629
MA	475327		0.8442	0.3331	0.8540	0.4038	0.9802	0.5808	0.2035	0.8467	0.5086	0.1879	0.8832
NH	141822		0.7274	0.5749	0.4319	0.2976	0.1712	0.0521	0.9395	0.8330	0.7322	0.6365	0.5456
NJ	179570		0.9857	0.7926	0.6116	0.4415	0.2815	0.1306	0.9881	0.8532	0.7255	0.6044	0.4893
NY	331589		0.0530	0.6964	0.3622	0.0482	0.7526	0.4740	0.2108	0.9619	0.7260	0.5023	0.2897
NC	353523		0.7841	0.4040	0.0476	0.7128	0.3977	0.1007	0.8201	0.5547	0.3032	0.0647	0.8381
PA	432879		0.4293	0.9638	0.5275	0.1175	0.7317	0.3680	0.0244	0.6994	0.3916	0.0995	0.8220
RI	68446		0.2815	0.2079	0.1389	0.0741	0.0131	0.9556	0.9013	0.8499	0.8012	0.7550	0.7112
SC	206236		0.8745	0.6528	0.4449	0.2496	0.0658	0.8925	0.7288	0.5739	0.4273	0.2881	0.1559
VT	85533		0.8511	0.7591	0.6729	0.5919	0.5157	0.4438	0.3759	0.3117	0.2509	0.1932	0.1383
VA	630560		0.0187	0.3406	0.7050	0.1079	0.5459	0.0160	0.5156	0.0422	0.5937	0.1682	0.7640
US	3615920		**8.5307**	**8.6426**	**6.9975**	**4.5733**	**6.3506**	**7.3120**	**9.4422**	**9.7276**	**8.1558**	**5.7159**	**9.3980**
		Unrepresented:	**255920**	**267920**	**223920**	**150920**	**215920**	**255920**	**339920**	**359920**	**309920**	**222920**	**375920**

Alabama Paradox

How is this possible?

State	House 299	House 300
AL	7.646	7.671
TX	9.640	9.672
IL	18.640	18.702

With the House size at 299, Alabama was the last state to be allotted an extra representative to make the House size because of it's decimal. When the House size was increased to 300, all states' quotas were increased by 0.33%. And there were two states that got the extra representatives; and, this time, Texas and Illinois beat out Alabama.

US Census Bureau

The U.S. Census Bureau is housed within the Department of Commerce.

Check out the U.S. Census Bureau for what it says about apportionment.

http://www.census.gov/

Summary 7-page brochure:
http://www.census.gov/prod/cen2010/briefs/c2010br-08.pdf

History of Legislation:

http://www.census.gov/history/www/reference/apportionment/apportionment_legislation_1790_-_1830.html

More!

For playing around, learning or teaching:

http://www.cut-the-knot.org/ctk/Democracy.shtml

Google

Google

US House apportionment

US History

The first proposed amendment to the US Constitution was called Article the First, also referred to as the Congressional Apportionment Amendment:

http://en.wikipedia.org/wiki/Article_the_First

Key Decades

The key decades in the history of the Congressional apportionment problem are 1790, 1840 and 1850, and 1920. Here are some excellent resources for each of these periods.

- Edmund J. James, *The First Apportionment of Federal Representatives in the United States*, Annals of the American Academy of Political and Social Science, 9 (January 1897): 1-41.
- Johanna Nicol Shields, *Whigs Reform the "Bear Garden": Representation and the Apportionment Act of 1842*, Journal of the Early Republic, 5 (Fall 1983): 356-82.
- Charles W. Eagles, *Democracy Delayed: Congressional Reapportionment and Urban-Rural Conflict in the 1920s*, University of Georgia Press, 1990.

US History

For any serious research of U.S. history, one must know about the Journals of Congress which includes the House Journal and the Senate Journal:

http://memory.loc.gov/ammem/amlaw/lwhj.html