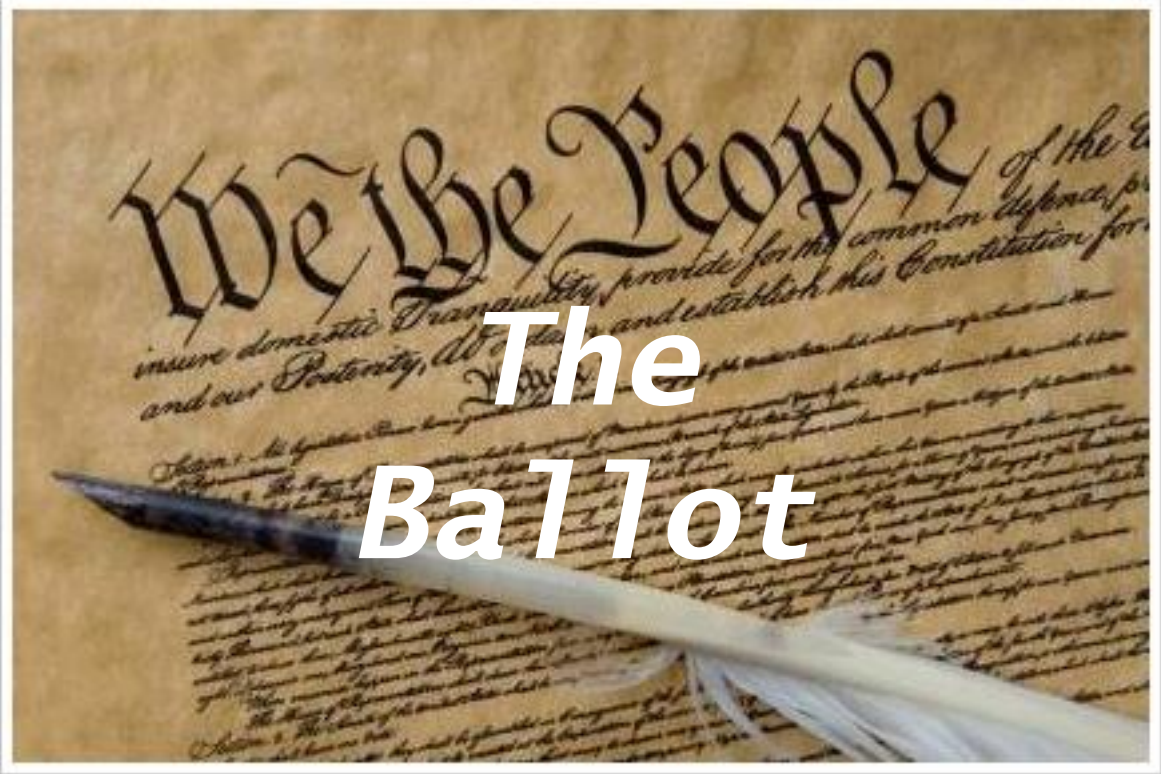


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MATH 5
Spring Semester 2017

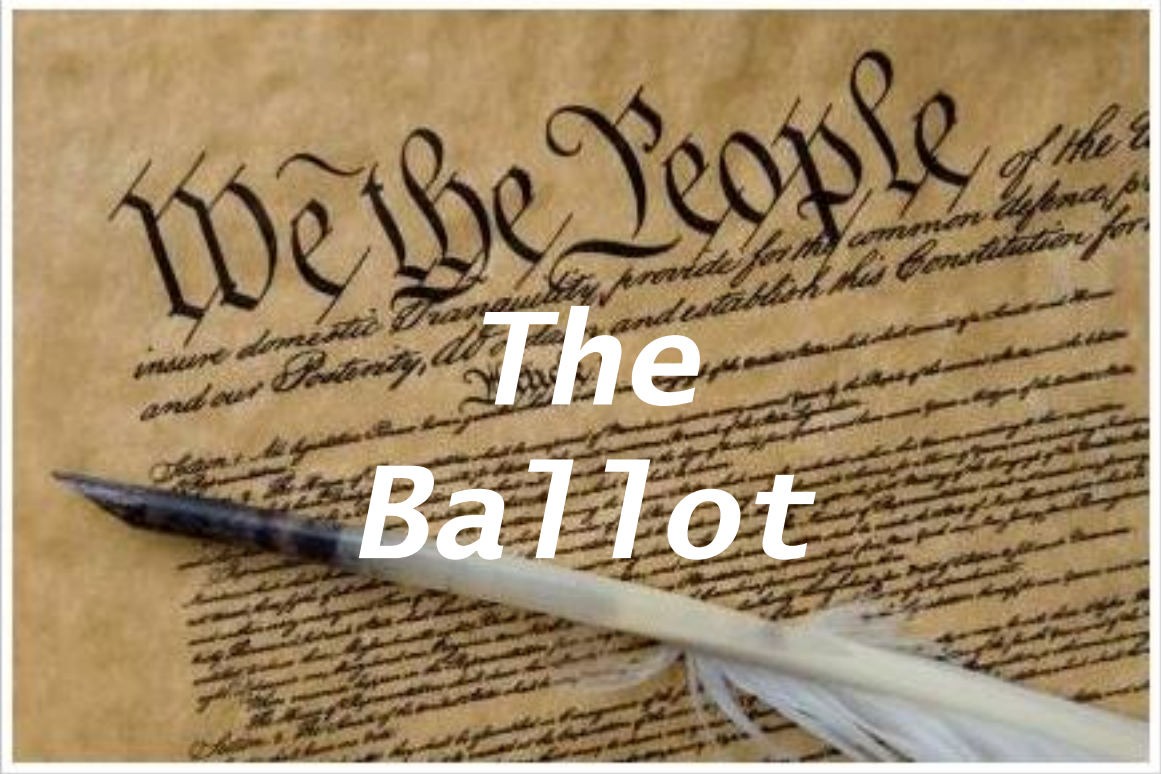


The Ballot

Advice

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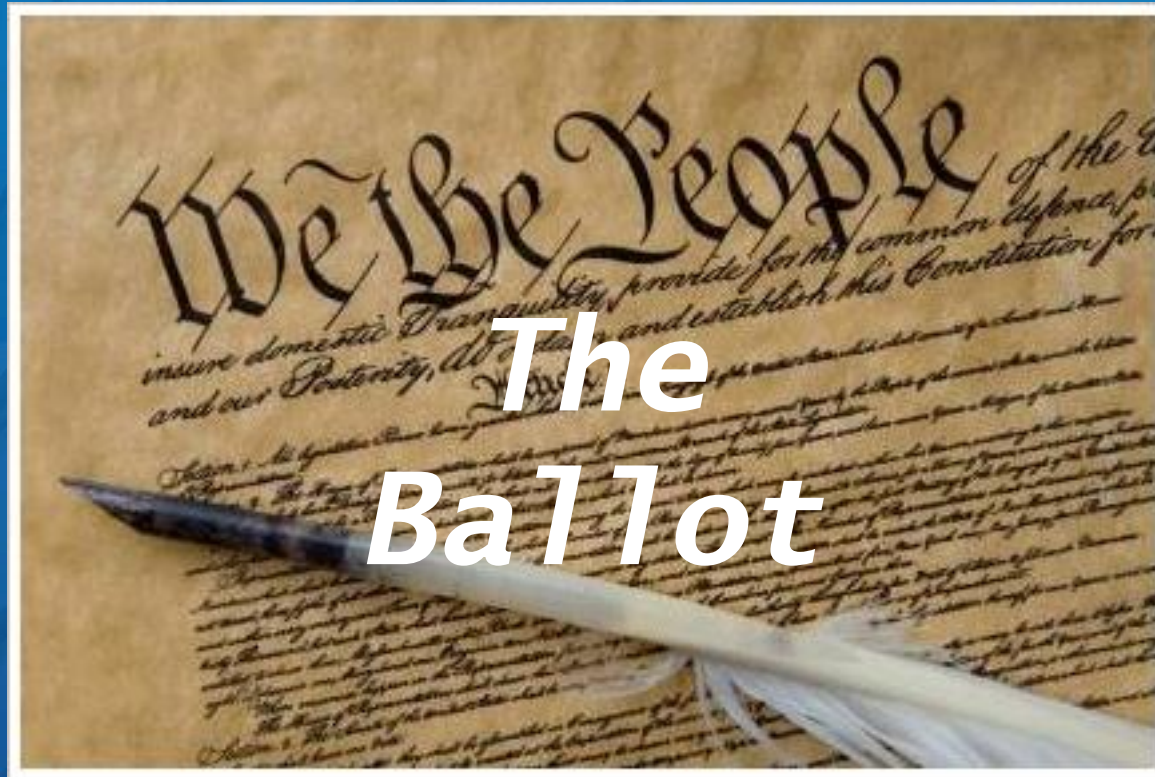


The Ballot

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Spring Semester 2017



Question



Did you follow the 2016
presidential election?



Question

What is an election?

Question

What is the difference between
a war and an election?

Question

A prime goal of democracy is to
replace wars with elections by
replacing bullets with ballots.

An *Election* is an example of a basic problem.

An *Election* is an example of a basic problem.

**How can one say something informative
about a group when the individuals in
the group are all different?**

Voters

*Who
wins?*

Ballots

The Ballot

An election must feature a ballot. We will assume the ballot is the same for each voter; further, one person/one ballot.

The Ballot

The structure of the ballot determines
your voice in an election.

The Ballot

During the Stalin era of the Soviet Union, a ballot looked like this:

The Ballot

During the Stalin era of the Soviet Union, a ballot looked like this:

District Commisar
Vote for one:

☐ Alesander Kolnovic

The Ballot

A two-option ballot looks like this:

Vote for One

☐ Option A

☐ Option B

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

- Majority Rule

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

- Majority Rule
- Unanimous

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

- Majority Rule
- Unanimous
- Super Majority

The Ballot

After the ballots are cast, the next step is to determine *who wins!*

- Majority Rule
- Unanimous
- Super Majority
- Minimum Threshold

A Multi-Option Ballot

A multi-option ballot looks like this:

Instruction

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

The Single Vote Ballot

A single-vote, multi-option ballot looks like this:

Vote for One

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

The Single Vote Ballot

A single-vote, multi-option ballot looks like this:

Vote for One

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

Notice how restricted the voter's voice is.

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- ☐ Option E

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Many voters will vote strategically rather than honestly.

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- ☐ Option E

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Who wins?

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- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

Notice how restricted the voter's voice is.

Many voters will vote strategically rather than honestly.

Who wins? Plurality

The Single Vote Ballot

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Vote for One

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

Notice how restricted the voter's voice is.

Many voters will vote strategically rather than honestly.

Who wins?

Plurality

Majority else top 2 run-off

Approval Voting

A multi-option ballot looks like this:

Approval List

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

Vote for all options
that you approve.

Approval Voting

A multi-option ballot looks like this:

Approval List

- ☐ Option A
- ☐ Option B
- ☐ Option C
- ☐ Option D
- ☐ Option E

Vote for all options
that you approve.

Who wins?

- Plurality
- Top 2 run-off if no majority
- Greatest majority

Approval Voting

A comparison. Here, $N = 100$.

Consider 100 voters in a three-way election:

- 26 first choice A but approve of B.
- 25 first choice A but approve neither B nor C.
- 15 first choice B but approve neither A nor C.
- 18 first choice C but approve of B.
- 16 first choice C but approve neither A nor B.

Approval Voting

A comparison. Here, $N = 100$.

Consider 100 voters in a three-way election:

- 26 first choice A
- 25 first choice A
- 15 first choice B
- 18 first choice C
- 16 first choice C

In a Vote for One election, A wins by majority rule.

Approval Voting

A comparison. Here, $N = 100$.

26	25	15	18	16
● A	● A	○ A	○ A	○ A
● B	○ B	● B	● B	○ B
○ C	○ C	○ C	● C	● C

vs. Vote for One option.

51	15	34
● A	○ A	○ A
○ B	● B	○ B
○ C	○ C	● C

Approval Voting

A comparison. Here, $N = 100$.

26	25	15	18	16
● A	● A	○ A	○ A	○ A
● B	○ B	● B	● B	○ B
○ C	○ C	○ C	● C	● C

A 51
B 59
C 34
B wins.

vs. Vote for One option.

51	15	34
● A	○ A	○ A
○ B	● B	○ B
○ C	○ C	● C

A 51
B 15
C 34
A wins.

Which Ballot?

Vote for One Party

- ☐ Hillary Clinton
and Tim Kaine
Democratic
- ☐ Gloria Estela La Riva
and Dennis J. Banks
Peace and Freedom
- ☐ Donald J. Trump
and Michael R. Pence
Republican, American Independent
- ☐ Gary Johnson
and Bill Weld
Libertarian
- ☐ Jill Stein
and Ajamu Baraka
Green

Which Ballot?

Vote for One Party

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- ☐ Donald J. Trump
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Republican, American Independent
- ☐ Gary Johnson
and Bill Weld
Libertarian
- ☐ Jill Stein
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Green

Vote for all Approved

- ☐ Hillary Clinton
and Tim Kaine
Democratic
- ☐ Gloria Estela La Riva
and Dennis J. Banks
Peace and Freedom
- ☐ Donald J. Trump
and Michael R. Pence
Republican, American Independent
- ☐ Gary Johnson
and Bill Weld
Libertarian
- ☐ Jill Stein
and Ajamu Baraka
Green

Ranked Choice Voting

In a ranked choice ballot the voter ranks some or all of the candidates.

In a top three system, you rank your top 3 choices as 1, 2, or 3.

In a full ranked system, if the ballot displays five choices, then you rank those choices 1 through 5.

The Ballot

In the 2010 Senate Race in California, consider these two ballots.

Vote for one:

- o Duane Roberts (G)
- o Marsha Feinland (PF)
- o Gail Lightfoot (L)
- o Barbara Boxer (D)
- o Carly Fiorina (R)
- o Edward Noonan (AI)

G = Green

PF = Peace & Freedom

L = Libertarian

AI = American Independent

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- ☐ Barbara Boxer (D)
- ☐ Carly Fiorina (R)
- ☐ Edward Noonan (AI)

G = Green

PF = Peace & Freedom

L = Libertarian

AI = American Independent

Rank each candidate once, 1 – 6:

	RANK	1	2	3	4	5	6
Duane Roberts		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Marsha Feinland		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Gail Lightfoot		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Barbara Boxer		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Carly Fiorina		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Edward Noonan		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

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Rank each candidate once, 1 – 6:

	RANK	1	2	3	4	5	6
Duane Roberts		<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Marsha Feinland		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
Gail Lightfoot		<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Barbara Boxer		<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Carly Fiorina		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
Edward Noonan		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>

A Background Story

XYZ Company employs 55 people. Health insurance legislation requires XYZ to provide insurance to its employees. XYZ receives bids from five companies: A, B, C, D and E. XYZ must adopt one of these plans which then applies to all its employees. All five plans are the same cost to XYZ; however, the benefits package differs between plans. Previously the CEO of XYZ sent a letter to its employees that XYZ would allow them to vote among the competing bids and that XYZ would honor their choice. XYZ holds an election and the employees vote their preferences.

The Ballots:

55 voters

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

The Ballots:

55 voters

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	D	C	B	C
Third	E			E	D	D
Fourth	C	C		B	C	B
Fifth	B			A	A	A

Who's the winner?

Majority Criterion

The Majority Criterion is considered to be a basic rule of fairness:

Majority Criterion

The Majority Criterion is considered to be a basic rule of fairness:

If a candidate gets a majority of votes, then that candidate should be declared the winner.

The Problem

If no candidate gets a majority of the votes, then is there such a thing as the “will of the people?”

The Problem

If no candidate gets a majority of the votes, then is there such a thing as the “will of the people?”

In general, can individual preferences be translated into a “group choice?”

Decision Mechanisms

Decision Mechanisms

- Plurality

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

The candidate with the most first place votes wins.

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E

The candidate with the most first place votes wins.

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E

The candidate with the most first place votes wins.

A wins.

Plurality

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N = 55

Plurality is the most common method in US elections.

Main drawback: the least desirable candidate may win. Here, the plurality winner A loses 37-18 in a one-on-one contest with any other candidate.

Decision Mechanisms

- Plurality
- Top two run-off

Top Two Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E

The two candidates with the most first place votes are A and B.

Top Two Run-Off

	18	12	10	9	4	2
A	A	B				
B			B		B	
				B		B
	B	A	A	A	A	A

Run-Off: A vs. B.

A: 18 votes.

B: 37 votes.

N=55, 28 needed to win.

Top Two Run-Off

	18	12	10	9	4	2
A		B				
			B		B	
				B		B
B		A	A	A	A	A

Run-Off: A vs. B.

A: 18 votes.

B: 37 votes.

B wins.

N=55, 28 needed to win.

Comedy Central

The Tea Party in Arizona is holding an endorsement convention. The nominees are Carly Fiorina, Ted Cruz, and Kevin McCarthy. There are 100 delegates who cast rank ordered ballots.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Fiorina (37) and McCarthy (32) are the top two.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina			McCarthy
McCarthy		Fiorina	McCarthy	
	McCarthy	McCarthy	Fiorina	Fiorina

Fiorina (37) and McCarthy (32) are the top two.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina			McCarthy
McCarthy		Fiorina	McCarthy	
	McCarthy	McCarthy	Fiorina	Fiorina

Fiorina (37) and McCarthy (32) are the top two.

Fiorina wins 53 – 47.

Comedy Central

27	10	16	15	32
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Now suppose that in the morning before the election, Carly Fiorina met with some of the delegates and gave an impressive performance to the McCarthy block and that two of the McCarthy supporters decided to switch to Fiorina.

Comedy Central

27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Now suppose that in the morning before the election, Carly Fiorina met with some of the delegates and gave an impressive performance to the McCarthy block and that two of the McCarthy supporters decided to switch to Fiorina.

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27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	McCarthy
McCarthy	Cruz	Fiorina	McCarthy	Cruz
Cruz	McCarthy	McCarthy	Fiorina	Fiorina

Now the run-off is between Fiorina and Cruz (39 – 31 – 30)!

Comedy Central

27 29	10	16	15	32 30
Fiorina	Fiorina	Cruz	Cruz	
	Cruz	Fiorina		Cruz
Cruz			Fiorina	Fiorina

Now the run-off is between Fiorina and Cruz (39 – 31 – 30)!

In the Fiorina vs. Cruz run-off, Cruz wins 61 – 39.

Comedy Central

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Fiorina	Fiorina	Cruz	Cruz	
	Cruz	Fiorina		Cruz
Cruz			Fiorina	Fiorina

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In the Fiorina vs. Cruz run-off, Cruz wins 61 – 39.

Increased support for Fiorina transforms her from a winner into a loser!

Decision Mechanisms

- Plurality
- Top two run-off
- Sequential elimination run-off

Sequential Elimination Run-Off

A top two run-off is a simplified elimination method. Along with plurality it is the most common method used in US political elections.

Some localities are replacing the top two run-off method with a ranked order system. For example, rank your top three choices.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Rank your top three choices

	<u>Rank</u>		
<u>Option</u>	1	2	3
A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
E	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Rank your top three choices

	<u>Rank</u>		
<u>Option</u>	1	2	3
A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
B	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
E	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D		
Second	D		B	C	B	C
Third		D			D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D
Third						

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C		B	C
Second			B	C		

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second			B			

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second			B			

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.
Still no majority. Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A		C	C		C
Second						

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.
Still no majority. Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A		C	C		C

N=55, 28 needed to win.

Rank your top three choices.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Still no majority. Eliminate D.
Still no majority. Eliminate B.

C wins 21-18.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the candidates

	<u>Rank</u>				
<u>Option</u>	1	2	3	4	5
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○
E	○	○	○	○	○

We now consider a full-ranked sequential elimination run-off.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the candidates

	<u>Rank</u>				
<u>Option</u>	1	2	3	4	5
A	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
B	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
C	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
D	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
E	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Rank the candidates

	<u>Rank</u>				
<u>Option</u>	1	2	3	4	5
A	○	○	○	○	●
B	○	●	○	○	○
C	○	○	○	●	○
D	○	○	●	○	○
E	●	○	○	○	○

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Eliminate the “least fit” candidate and then recount the votes.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D		
Second	D		B	C	B	C
Third		D			D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D
Third	C	C	D	B	C	B
Fourth	B	A	A	A	A	A
Fifth						

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	B	C
Second	D	D	B	C	D	D
Third	C	C	D	B	C	B
Fourth	B	A	A	A	A	A
Fifth						

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C		B	C
Second			B	C		
Third	C	C		B	C	B
Fourth	B	A	A	A	A	A
Fifth						

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second	C	C	B	B	C	B
Third	B	A	A	A	A	A
Fourth						
Fifth						

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	C	B	C
Second	C	C	B	B	C	B
Third	B	A	A	A	A	A
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A		C	C		C
Second	C	C			C	
Third		A	A	A	A	A
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	C	C	C	C	C
Second	C	A	A	A	A	A
Third						
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	C	C	C	C	C
Second	C	A	A	A	A	A
Third						
Fourth						
Fifth						

N=55, 28 needed to win.

Eliminate the “least fit” candidate and then recount the votes.

The candidate with the least first place votes is E; hence eliminate E.

Next eliminate D.

Next eliminate B.

C wins: 37-18 !

News Flash In the 2016 election Maine approved a full ranked system for statewide offices.

[https://ballotpedia.org/Maine_Ranked_Choice_Voting_Initiative, Question_5_\(2016\)](https://ballotpedia.org/Maine_Ranked_Choice_Voting_Initiative, Question_5_(2016))

Sequential Elimination Run-Off

The full sequential elimination method is used by the International Olympic Committee to decide the site of the Olympic Games.

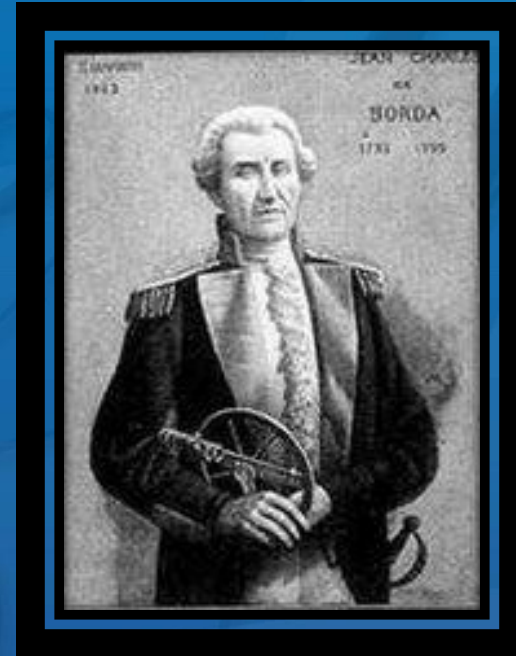
In local political elections, “least fit” sequential elimination is more commonly known as Instant Runoff Voting (IRV).

Decision Mechanisms

- Plurality
- Top two run-off
- Sequential elimination run-off
- Borda Count

Borda Count

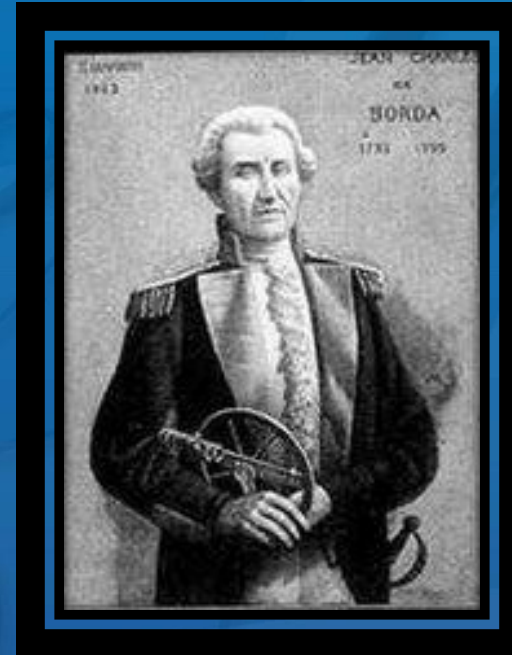
	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A



Jean-Charles
Chevalier de Borda
1733 - 1799

Borda Count

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A



Jean-Charles
Chevalier de Borda
1733 - 1799

First: 5 points.
Second: 4 points.
Third: 3 points.
Fourth: 2 points.
Fifth: 1 point.

Borda Count

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A: 127

B: 156

C: 158

D: 191

E: 146

Borda Count



	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A: 127 A: 72
B: 156 B: 101
C: 158 C: 117
D: 191 D: 136
E: 146 E: 92

Borda Count



	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A: 127 A: 72
 B: 156 B: 101
 C: 158 C: 117
 D: 191 D: 136
 E: 146 E: 92

D wins!

Borda Count

The Borda Count is used in these situations:

- Heisman Trophy
- AL and NL MVP awards
- Country Music Vocalist of the year.

Borda Count

Problem: The Borda Count can violate the Majority Criterion of Fairness.

Borda Count

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	6	2	3
First	A	B	C
Second	B	C	D
Third	C	D	B
Fourth	D	A	A

Borda Count

Problem: The Borda Count can violate the Majority Criterion of Fairness.

	6	2	3
First	A	B	C
Second	B	C	D
Third	C	D	B
Fourth	D	A	A

Borda Points:

A: 29

B: 32

C: 30

D: 19

B wins!

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

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A rank-order ballot is taken and the class decides on chocolate.

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The teacher goes out for the ice cream and brings back vanilla.

Comedy Central

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A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

The students ask, Why vanilla? Were they out of chocolate?

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

The students ask, Why vanilla? Were they out of chocolate?

The teacher says, Well, no. They had chocolate. I got vanilla because they were out of strawberry.

Comedy Central

The teacher offers to buy the class ice cream if the students will agree on one flavor. The choices are strawberry, chocolate, and vanilla.

A rank-order ballot is taken and the class decides on chocolate.

The teacher goes out for the ice cream and brings back vanilla.

The students ask, Why vanilla? Were they out of chocolate?

The teacher says, Well, no. They had chocolate. I got vanilla because they were out of strawberry.

How could this happen?

Comedy Central

4	2	5
Strawberry	Vanilla	Chocolate
Vanilla	Chocolate	Strawberry
Chocolate	Strawberry	Vanilla

Comedy Central

4	2	5
Strawberry	Vanilla	Chocolate
Vanilla	Chocolate	Strawberry
Chocolate	Strawberry	Vanilla

Plurality, top two run-off, IRV, and the Borda Count
all produce Chocolate as the winner.

Comedy Central

4	2	5
Strawberry	Vanilla	Chocolate
Vanilla	Chocolate	Strawberry
Chocolate	Strawberry	Vanilla

But since they were out of strawberry, then the teacher eliminated strawberry from the count.

Comedy Central

4	2	5
Vanilla Chocolate	Vanilla Chocolate	Chocolate Vanilla

But since they were out of strawberry, then the teacher eliminated strawberry from the count.

So, Vanilla wins 6 – 5!

Decision Mechanisms

- Plurality
- Top two run-off
- Sequential elimination run-off
- Borda Count
- Condorcet

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.



Marie Jean Antoine
Nicholas de Caritat,
Marquis de Condorcet
1743 - 1794

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.



Marie Jean Antoine
Nicholas de Caritat,
Marquis de Condorcet
1743 - 1794

Note: A is a Condorcet Loser!

Condorcet

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

If there is a candidate that can beat all rivals one-on-one, then that candidate is the winner.

A vs. E
18 37

B vs. E
22 33

C vs. E
19 36

D vs. E
27 28

E wins!

The Winner

- Plurality
- Top two run-off
- IRV Sequential elimination
- Borda Count
- Condorcet

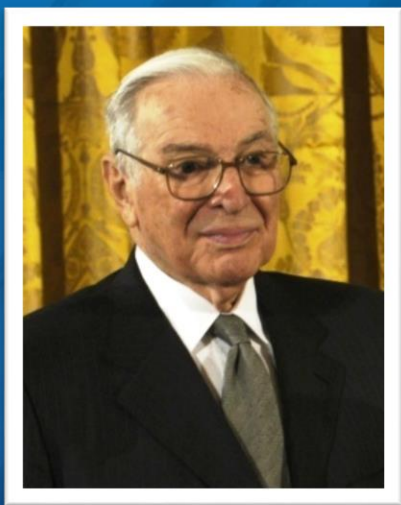
The Winner

- Plurality: A
- Top two run-off: B
- IRV Sequential elimination: C
- Borda Count: D
- Condorcet: E

The Question

Is there a fair way to decide?

Arrow's Theorem



Kenneth Arrow
Nobel Prize in Economics 1972

There is no voting system that
can satisfy all basic requirements
of fairness in all cases.

Fairness Axioms

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- Individual Sovereignty (free ballot)

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- Individual Sovereignty (free ballot)
- Majority Rule

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- An increase in support does not damage one's chance of winning.

Fairness Axioms

- Individual Sovereignty (free ballot)
- Majority Rule
- Condorcet Rule
- An increase in support does not damage one's chance of winning.
- If a losing candidate drops out and voter preferences are constant, then the election result is not changed.

Condorcet Paradox

A simple way to illustrate the difficulty is to consider the following example.

Voter 1: A, B, C

Voter 2: B, C, A

Voter 3: C, A, B

Condorcet Paradox

A simple way to illustrate the difficulty is to consider the following example.

Voter 1: A, B, C

Voter 2: B, C, A

Voter 3: C, A, B

In pairwise voting,

A beats B, 2 to 1;

B beats C, 2 to 1;

yet, C beats A, 2 to 1.

Pairwise voting may not be transitive.

Points to Ponder

Even though “perfection” isn’t possible,
improvement is possible.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

A is the Vote for One winner.

C is the least fit IRV winner.

E is the Condorcet winner.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

Coombs: Eliminate the “least desirable” candidate and then recount the votes.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	A	B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B	A	A	A	A	A

N=55, 28 needed to win.

Coombs: Eliminate the “least desirable” candidate and then recount the votes.

Eliminate A.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First		B	C	D	E	E
Second	D	E	B	C	B	C
Third	E	D	E	E	D	D
Fourth	C	C	D	B	C	B
Fifth	B					

N=55, 28 needed to win.

Coombs: Eliminate the “least desirable” candidate and then recount the votes.

Eliminate A.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	B	C	D	E	E
Second	E	E	B	C	B	C
Third	C	D	E	E	D	D
Fourth	B	C	D	B	C	B

Coombs: Eliminate the “least desirable” candidate and then recount the votes.

Eliminate A.

N=55, 28 needed to win.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	B	C	D	E	E
Second	E	E	B	C	B	C
Third	C	D	E	E	D	D
Fourth	B	C	D	B	C	B

N=55, 28 needed to win.

Coombs: Eliminate the
“least desirable”
candidate and then
recount the votes.

Eliminate A.

Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	E	C	D	E	E
Second	E	D	E	C	D	C
Third	C	C	D	E	C	D

N=55, 28 needed to win.

Coombs: Eliminate the
“least desirable”
candidate and then
recount the votes.

Eliminate A.
Eliminate B.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	E	C	D	E	E
Second	E	D	E	C	D	C
Third	C	C	D	E	C	D

N=55, 28 needed to win.

Coombs: Eliminate the
“least desirable”
candidate and then
recount the votes.

Eliminate A.
Eliminate B.
Eliminate C.

Sequential Elimination Run-Off

	18	12	10	9	4	2
First	D	E	E	D	E	E
Second	E	D	D	E	D	D

N=55, 28 needed to win.

D: 27 votes

E: 28 votes

E wins!

Coombs: Eliminate the “least desirable” candidate and then recount the votes.

Eliminate A.

Eliminate B.

Eliminate C.

The Winner

- Plurality: A
- Top two run-off: B
- IRV Sequential elimination: C; **Coombs: E**
- Borda Count: D
- **Condorcet: E**

Note: “least desirable” sequential elimination (Coombs Method) for a fully ranked ballot will always select the Condorcet winner if there is one.

The Ballot

Structure

- **List Only**
 - Vote for One
 - Vote for Approved
- **List and Rank**
 - Partial
 - Full

Decision

- **List Only**
 - Plurality
 - Run-Off
- **List and Rank**
 - IRV
 - Coombs
 - Borda
 - Condorcet

Case Study

The Australian Electoral System

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets $\frac{1}{2}$; son #2 gets $\frac{1}{3}$; son #3 gets $\frac{1}{9}$.



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The father dies. The sons are in a quandary.



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The father dies. The sons are in a quandary.

$$\frac{1}{2} \times 17 = 8 \frac{1}{2}$$

$$\frac{1}{3} \times 17 = 5 \frac{2}{3}$$

$$\frac{1}{9} \times 17 = 1 \frac{8}{9}$$

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A wise man notes the dilemma of the sons and contributes his camel, yielding 18 camels to be distributed.

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets $\frac{1}{2}$; son #2 gets $\frac{1}{3}$; son #3 gets $\frac{1}{9}$.

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$$\frac{1}{9} \times 17 = 1 \frac{8}{9}$$

$$\frac{1}{2} \times 18 = 9$$

$$\frac{1}{3} \times 18 = 6$$

$$\frac{1}{9} \times 18 = 2$$

Each son gets more than his inheritance.

Comedy Central

Recently in Egypt a father with 17 camels willed his estate to his sons as follows: son #1 gets $\frac{1}{2}$; son #2 gets $\frac{1}{3}$; son #3 gets $\frac{1}{9}$.

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$$\frac{1}{3} \times 17 = 5 \frac{2}{3}$$

$$\frac{1}{9} \times 17 = 1 \frac{8}{9}$$

$$\frac{1}{2} \times 18 = 9$$

$$\frac{1}{3} \times 18 = 6$$

$$\frac{1}{9} \times 18 = 2$$

Each son gets more than his inheritance.

And the wise man rides off on the left over camel.





Problems

In case of a tie in a political election:

<http://fivethirtyeight.com/datalab/the-2014-elections-that-ended-in-a-tie/>

<http://www.theatlantic.com/politics/archive/2012/11/when-a-state-election-can-be-literally-determined-by-a-coin-toss/265413/>