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Humboldt State University
Mathematics Colloquium Spring 2017


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## Question

What is an election?

## Question

## What is the difference between a war and an election?

## Question

A prime goal of democracy is to replace wars with elections by replacing bullets with ballots.

## An Election is an example of a basic problem.

An Election is an example of a basic problem.
How can one say something informative about a group when the individuals in the group are all different?

## Voters

## Ballots

$$
\begin{aligned}
& \text { Who } \\
& \text { wins? }
\end{aligned}
$$

## Ballots

## What do you see?



## Two Models

> Electoral College Model
$>$ States Model

## Electoral College Model



## States Model



## The States Model

In a presidential election the electorate consists of the 50 states plus the District of Columbia.

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In a presidential election the electorate consists of the 50 states plus the District of Columbia.

The ballot is not one-state, one-vote.
The ballot is a weighted ballot as described by the electoral system.

## The Electoral College

The President is elected by a majority vote of the electors as specified by the U. S. Constitution, Article II, Section 1 with Amendment XII (ratified in 1804) and Amendment XX (ratified in 1933).

## The Electoral College

The College consists of a slate of electors from each state. The number of electors equals the number of members of Congress-the number of representatives in the House plus two senators.

Amendment XXIII (ratified 1961) allows the District of Columbia three electors.


## Apportionment of the U.S. House of Representatives Based on the 2010 Census



## Small State Bias

The Electoral College is heavily weighted towards smaller states.

California has 66 times the population of Wyoming. The electoral vote ratio is CA 55 and WY 3.

# Background for <br> Congressional Apportionment 

## Two Math Skills

1. Average two different positive numbers.
2. Round a positive decimal number.

## Two Math Skills

1. The average of $a$ and $b$ where $0<a<b$.

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$$
\begin{aligned}
\operatorname{ave}(a, b)=\max (a, b) & =b \\
\min (a, b) & =a \\
\operatorname{AM}(a, b) & =(a+b) / 2 \\
\operatorname{GM}(a, b) & =\sqrt{a \times b} \\
\operatorname{HM}(a, b) & =\frac{2}{\frac{1}{a}+\frac{1}{b}}=\frac{2 a b}{a+b}
\end{aligned}
$$

## Two Math Skills

For Example: ave $(8,12)=$
$>\max (8,12)=12$
$>\min (8,12)=8$
$>A M(8,12)=10$
$>\operatorname{GM}(8,12)=\sqrt{8 \times 12} \approx 9.8$
$>\mathrm{HM}(8,12)=\frac{2}{\frac{1}{8}+\frac{1}{12}}=9.6$

## Two Math Skills

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$>\mathrm{HM}(8,12)=\frac{2}{\frac{1}{8}+\frac{1}{12}}=9.6$
Always, $\min <\mathrm{HM}<\mathrm{GM}<\mathrm{AM}<\max$.

## Two Math Skills

2. How to round a positive decimal number.

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Suppose $q$ is a positive decimal number, not an integer, whose integer part is $n$ so $n<q<n+1$.

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## Two Math Skills

2. How to round a positive decimal number.

Suppose $q$ is a positive decimal number, not an integer, whose integer part is $n$ so $n<q<n+1$.
Then $\operatorname{round}(q)$ is either $n$ or $n+1$ where $\operatorname{round}(q)=n+1$ if and only if $q \geq$ ave $(n, n+1)$ :

$$
\begin{aligned}
q \geq \max (n, n+1) & \text { round down } \\
\min (n, n+1) & \text { round up } \\
\operatorname{AM}(n, n+1) & \text { round normally } \\
\mathrm{GM}(n, n+1) & \text { geometric mean } \\
\operatorname{HM}(n, n+1) & \text { harmonic mean }
\end{aligned}
$$

## Congressional Apportionment Two Approaches

- Constituency Approach:
- House Size Approach:


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- House Size Approach:


## Congressional Apportionment Two Approaches

- Constituency Approach: How many people should a congressperson represent?
- House Size Approach: How many seats should there be in the House?


## Basic Divisor Period

## 1790-1840

## Basic Divisor Methods

Step 1: How many should a congressman represent?
Step 2: Divide into a state's population.
Step 3: Round the resulting decimal.

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1790-1830: round down
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Also proposed:
round up
round by harmonic mean

## Quota Period

## 1850-1900

## Quota Method

Step 1. Decide House size, $h$.
Step 2. Calculate each state's quota:

$$
\text { Quota }=(\text { House size }) \times \frac{\text { state population }}{\text { national population }}
$$

Let $n=$ integer part of the Quota.
Step 3. Select $a \in\{n, n+1\}$ so that $h$ is attained.

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First give each state its lower quota.
Use the decimal part as a priority list.

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First give each state its lower quota. Hamilton's
Use the decimal part as a priority list.
Method

## Alabama Paradox

When the number of House seats is increased, a given state's apportion may decrease.

## The Deal Breaker

Results from the 1890 census doomed Hamilton's Method.


# Modified Divisor Period 

1910 - present

# Ad-hoc Modified Divisor 

Step 1. Decide the House size: $h$.
Step 2. Apply a basic divisor method to obtain the preset $h$.

## Priority

Step 1. Give one seat to each state.
Step 2. Construct a sorted list of priority numbers.
Step 3. Award seats one at a time by priority until the desired House size is reached.

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& A_{n}=\frac{\text { state population }}{\text { ave }(n, n+1)} \\
& A_{n}=\frac{\text { state population }}{\sqrt{n \times(n+1)}}
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## The Aftermath

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There are no perfect apportionment methods.

Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

## The Presidential Election

In each state except Maine and Nebraska, the electoral slate is decided by plurality winner of the popular vote, known as "winner take all."

When you cast a vote for candidate $X$ in a presidential election, you are casting a vote for X's slate of electors in your state.

## The 2016 Presidential Election

The popular vote:

| Hillary Clinton: | $65,844,610$ | $48.2 \%$ |
| :--- | ---: | ---: |
| Donald Trump: | $62,979,636$ | $46.1 \%$ |
| Others: | $7,804,213$ | $5.7 \%$ |

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The Electoral College vote:
Hillary Clinton: 227
Donald Trump: 304
Others: 7

## The Popular Vote

Arguably, on 4 other occasions in U. S. history the electoral and popular systems produced different results.

1. John Quincy Adams vs. Andrew Jackson 1824
2. Rutherford B. Hayes vs. Samuel Tilden 1876
3. Benjamin Harrison vs. Grover Cleveland 1888
4. George Bush vs. Al Gore 2000

## 1876

| Candidate | Party | Popular Vote | Electoral Vote |
| :--- | :---: | :---: | :---: |
| Rutherford B. Hayes (OH) | Republican | $4,034,142$ | 185 |
| Samuel J. Tilden (NY) | Democratic | $4,286,808$ | 184 |
| Peter Cooper (NY) | Greenback | 83,726 | --- |

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Lesson: Hayes' Electoral College victory was an artifact of the method used for congressional apportionment.

The original apportionment based on the 1870 census used the Hamilton Quota Method. The 1872 supplement act added nine seats but used a different method. The methods agreed except for two seats: the original method would have awarded the seats to Illinois and New York, but the supplement awarded them to New Hampshire and Florida.

## 2000

| Candidate | Party | Popular Vote | Electoral Vote |
| :--- | :---: | :---: | :---: |
| George W. Bush (TX) | Republican | $5,443,633$ | 271 |
| Albert Gore (TN) | Democratic | $5,538,163$ | 266 |
| Ralph Nader (DC) | Green | 250,017 | --- |
| Patrick Buchanan (VA) | Reform | 149,115 | --- |

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Lesson: Bush's Electoral College victory was an artifact of the size of the House of Representatives.

## Neubauer and Zeitlin

Neubauer and Zeitlin calculated the Electoral College vote based on House sizes 50-1000 using the current method of congressional apportionment.

For all House sizes larger than 597, except 655 which produces a tie, Gore wins. For all House sizes smaller than 491 Bush wins.

## Neubauer and Zeitlin

In the intermediate range 492-597 the winner oscillates between Bush and Gore.

For these 106 House sizes, there are 24 ties, Bush wins 53 times, and Gore wins 29 times.

## http://www.thirty-thousand.org/pages/Neubauer-Zeitlin.htm

Also, Michael Neubauer and Joel Zeitlin, Apportionment and the 2000 Election, The College Mathematics Journal 34(1), January 2003: 2-10.

## 2000

The 2000 election displays another potential problem. The number of electoral votes each state gets is tied to the decennial census. Although the election was in 2000, apportionment of the House was based on the 1990 census. An election held in a census year is based on a population that is ten years old.

What would have been the result of Bush vs. Gore if the Electoral College vote were based on the 2000 census for congressional apportionment?

## 2000

In comparison with the 1990 census the 2000 census affected congressional apportionment for 18 states shifting 12 seats:

Arizona, gain 2; California, gain 1; Colorado, gain 1; Connecticut, lose 1; Florida, gain 2; Georgia, gain 2; Illinois, lose 1; Indiana, lose 1; Michigan, lose 1; Mississippi, lose 1; Nevada, gain 1; New York, lose 2; North Carolina, gain 1; Ohio, lose 1; Oklahoma, lose 1; Pennsylvania, lose 2; Texas, gain 2; Wisconsin, lose 1.

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Accordingly, the electoral vote would have changed from
Bush 271 and Gore 266
to
Bush 277 and Gore 259.

## Reform

Over the past 200 years, over 700 proposals have been introduced in Congress to reform or eliminate the Electoral College. There have been more proposals for Constitutional amendments on changing the Electoral College than on any other subject.
https://www.archives.gov/federal-register/electoral-college/faq.html\#whyec

## Today's Debate

Resolved: The electoral system should be replaced by the popular vote system.

What's the Popular Vote System?

## The 2016 Presidential Election

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## Arrow's Theorem



There is no voting system that can satisfy basic requirements of fairness in all cases.

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## Kenneth Arrow

Nobel Prize in Economics 1972

> | https://www.washingtonpost.com/national/kenneth-arrow-nobel-laureate-and- |
| :--- |
| seminal-economist-with-wide-impact-dies-at-95/2017/02/21/089c3888-f8aa- |
| 11e6-be05-1a3817ac21a5 story.html?hpid=hp hp-more-top-stories ob-main- |
| arrow-1017pm\%3Ahomepage\%2Fstory\&utm term=.bcb13cdccd94 |

## Fairness Axioms

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- Individual Sovereignty


## Question 1

## Should there be a uniform national presidential ballot?

$>$ Should there be a national ballot access law?

## Question 1

## Should there be a uniform national presidential ballot?

> Should there be a national ballot access law? Ballots differ state to state. GA, IN, OK listed 3 candidates; CA 5; TN 7; UT 10; CO 22.

## Question 1

## Should there be a uniform national presidential ballot?

> Should there be a national ballot access law?
$>$ Should there be a write-in provision?
Today 9 states do not allow a write-in.
https://ballotpedia.org/Ballot access for presidential candidates

## Question 2

Should there be uniform suffrage for a national popular vote?

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$>$ Should all American citizens "in goodstanding" be allowed to vote in the national popular election for President?
http://felonvoting.procon.org/view.resource.p
hp?resourcelD=000286

## Question 2

Should there be uniform suffrage for a national popular vote?
$>$ Should all American citizens "in goodstanding" be allowed to vote in the national popular election for President?
> What about American citizens who live in a U.S. territory but are not citizens of a state or residents of D. C.?

## Question 3

What should be the structure of the ballot in a national presidential election?

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> Vote for One.

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$>$ Vote for One.
$>$ Approval.

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What should be the structure of the ballot in a national presidential election?
> Vote for One.
$>$ Approval.
$>$ Ranked Choice.

## The Ballot

An election must feature a ballot. Assume the ballot is the same for each voter; further, one person/one ballot.

## The Ballot

## The structure of the ballot determines

 your voice in an election.
## The Ballot

## During the Stalin era of the Soviet Union, a ballot looked like this:

## The Ballot

## During the Stalin era of the Soviet Union, a ballot looked like this:

District Commisar Vote for One<br>Alesander Kolnovic

## The Ballot

## A two-option ballot looks like this:

| Vote for One |
| :--- |
| $\square$ Option A |
| $\square$ Option B |

## The Single Vote Ballot

A multi-option ballot looks like this:

| Instruction |  |
| :--- | :--- |
| $\square$ | Option A |
| $\square$ | Option B |
| $\square$ | Option C |
| $\square$ | Option D |
| $\square$ | Option E |

## The Single Vote Ballot

A multi-option ballot looks like this:

| Vote for One |
| :--- |
| $\square$ |
| Option A |
| $\square$ |
| Option B |
| $\square$ |
| Option C |
| $\square$ Option D |
| $\square$ Option E |

## The Single Vote Ballot

A multi-option ballot looks like this:

| Vote for One |
| :--- |
| $\square$ |
| Option A |
| $\square$ |
| Option B |
| $\square$ Option C |
| $\square$ Option D |
| $\square$ Option E |

Notice how restricted the voter's voice is.

## The Single Vote Ballot

A multi-option ballot looks like this:

| Vote for One |
| :--- |
| $\square$ Option A |
| $\square$ Option B |
| $\square$ Option C |
| $\square$ Option D |
| $\square$ Option E |

Notice how restricted the voter's voice is.

Many voters vote
strategically rather than honestly.

## Approval Voting

A multi-option ballot looks like this:

| Approval List |  |
| :--- | :--- |
| $\square$ Option A | Vote for all options |
| $\square$ Option B | that you approve. |
| $\square$ Option C |  |
| $\square$ Option D |  |
| $\square$ Option E |  |

## Which Ballot?



## Which Ballot?



## Vote for all Approved

Hillary Clinton and Tim Kaine
Democratic
Gloria Estela La Riva and Dennis J. Banks
Peace and Freedom
$\square$ Donald J. Trump
and Michael R. Pence
Republican, American Independent
Gary Johnson
and Bill Weld
Libertarian
$\square$ Jill Stein
and Ajamu Baraka
Green

## Ranked Choice Voting

In a ranked choice ballot the voter ranks some or all of the candidates.

In a full ranked system, if the ballot displays five choices, then you rank those choices 1 though 5.

News Flash In the 2016 election Maine approved a full ranked system for statewide offices.<br>https://ballotpedia.org/Maine Ranked Choice Voting Initiative, Question 5 (2016)



## Instant Runoff Voting

| Rank the options |  |
| :---: | :---: |
|  | Rank |
| Option | 12345 |
| A | $\bigcirc \bigcirc \bigcirc \bigcirc 0$ |
| B | $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ |
| C | $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ |
| D | $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ |
| E | $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ |

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |  |
| Second | D | E | B | C | B | C |  |
| Third | E | D | E | E | D | D |  |
| Fourth | C | C | D | B | C | B |  |
| Fifth | B | A | A | A | A | A |  |
| N $=55,28$ needed to win. |  |  |  |  |  |  |  |


| Rank the options |  |
| :---: | :---: |
|  | Rank |
| Option | 12345 |
| A | $\bigcirc \bigcirc 000$ |
| B | $\bigcirc \bigcirc \bigcirc \bigcirc$ |
| C | $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ |
| D | $\bigcirc \bigcirc \bigcirc 000$ |
| E | $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ |

## Who Wins?

## Vote for One



Vote for One: The candidate with the most first place votes wins.

## Vote for One



Vote for One:
The candidate
with the most
first place votes
wins.

A wins.

## Vote for One

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Ffith | B | A | A | A | A | A |

Problem: the least desirable candidate may win.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Fifth | B | A | A | A | A | A |

No candidate gets a majority of first place votes.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Fifth | B | A | A | A | A | A |

Eliminate the "least fit" candidate and then recount the votes.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
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Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D |  |  |  |  |  |  |
| Second | D |  | B | C | B | C |  |  |  |  |
| Third |  | D |  |  | D | D |  |  |  |  |
| Fourth | C | C | D | B | C | B |  |  |  |  |
| Ffith | B | A | A | A | A | A |  |  |  |  |
| N $=55,28$ needed to win. |  |  |  |  |  |  |  |  |  |  |

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|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | B | C |
| Second | D | D | B | C | D | D |
| Third | C | C | D | B | C | B |
| Fourth | B | A | A | A | A | A |
| Fifth |  |  |  |  |  |  |

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | B | C |
| Second | D | D | B | C | D | D |
| Third | C | C | D | B | C | B |
| Fourth |  |  |  |  |  |  |
| Fifth |  |  |  |  |  |  | B 1 A $A$ A $A$

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C |  | B | C |
| Second |  |  | B | C |  |  |
| Third | C | C |  | B | C | B |
| Fourth | B | A | A | A | A | A |
| Fifth |  |  |  |  |  |  |

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.<br>Next eliminate D.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | C | B | C |
| Second | C | C | B | B | C | B |
| Third | B | A | A | A | A | A |
| Fourth |  |  |  |  |  |  |
| Fifth |  |  |  |  |  |  |

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | C | B | C |
| Second | C | C | B | B | C | B |
| Third | B | A | A | A | A | A |
| Fourth |  |  |  |  |  |  |
| Fifth |  |  |  |  |  |  |

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.
Next eliminate B.

## Instant Runoff Voting

$\left.\begin{array}{|l|c|c|c|c|c|c|}\hline & 18 & 12 & 10 & 9 & 4 & 2 \\ \hline \text { First } & \text { A } & & \text { C } & \text { C } & & \text { C } \\ \text { Second } \\ \text { Third } \\ \text { Fourth } \\ \text { Fifth }\end{array}\right]$

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.
Next eliminate B.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | C | C | C | C | C |
| Second |  |  |  |  |  |  |
| Third |  |  |  |  |  |  |
| Fourth |  |  |  |  |  |  |
| Fifth |  |  |  |  |  |  |$|$| C | A |  |  |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| A | A |  |  |  |  |  |  |
| A |  |  |  |  |  |  |  |
| $\mathrm{N}=55,28$ needed to win. |  |  |  |  |  |  |  |

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.
Next eliminate B.

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | C | C | C | C | C |
| Second |  |  |  |  |  |  |
| Third |  |  |  |  |  |  |
| Fourth |  |  |  |  |  |  |
| Fifth |  |  |  |  |  |  |$|$| A | A |  |  |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| A | A |  |  |  |  |  |  |
| A |  |  |  |  |  |  |  |
| $\mathrm{N}=55,28$ needed to win. |  |  |  |  |  |  |  |

Eliminate the "least fit" candidate and then recount the votes. Eliminate E.

Next eliminate D.
Next eliminate B.
C wins: 37-18!

## Instant Runoff Voting

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Fifth | B | A | A | A | A | A |
| N $=55,28$ needed to win. |  |  |  |  |  |  |

Question: How many votes did C get?

## Fairness Axioms

- Individual Sovereignty
- Majority Rules
- Condorcet Rule


## Condorcet

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Fifth | B | A | A | A | A | A |

If a candidate can beat all rivals one-on-one, then that candidate is the winner.

## Condorcet

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Fifth | B | A | A | A | A | A |

If a candidate can beat all rivals one-on-one, then that candidate is the winner.

| A vs. E | B vs. E | C vs. E | D vs. E |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 37 | 22 | 33 | 19 | 36 | | 27 | 28 |
| :--- | :--- |$\quad$ wins!

## Condorcet

|  | 18 | 12 | 10 | 9 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| First | A | B | C | D | E | E |
| Second | D | E | B | C | B | C |
| Third | E | D | E | E | D | D |
| Fourth | C | C | D | B | C | B |
| Fifth | B | A | A | A | A | A |

Coombs: Eliminate the least desirable candidate, the candidate with the most last place votes.

| A vs. E | B vs. E | C vs. E | D vs. E |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 37 | 22 | 33 | 19 | 36 | | 27 | 28 |
| :--- | :--- |$\quad$ E wins!

## State Sovereignty

Or, we could just keep voting as a "state's rights" matter. Ballot access, ballot structure, suffrage, voting mechanics would be left up to each state. Then count the certified popular vote in each state as is currently done.

## State Sovereignty

Or, we could just keep voting as a "state's rights" matter. Ballot access, ballot structure, suffrage, voting mechanics would be left up to each state. Then count the certified popular vote in each state as is currently done.

What could possibly go wrong?

Should the electoral system be replaced by a popular vote system?

## Thank You

## What is an election?

http://www.nia977.wix.com/drbcap

