Congressional Apportionment

Bringing Down the House

Charles Biles, Ph.D.
Mathematics 103i: Contemporary Mathematics
Humboldt State University
Fall Semester 2017

website: nia977.wix.com/drbcap

> "… no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, …"
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The Apportionment Problem

Determine how many seats in the U.S. House of Representatives each state gets.

Apportionment History

Last time we looked at the first two periods of congressional apportionment history.

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The Basic Divisor Method: 1790-1840.

The Quota Method: 1850-1900.

1910

Apportionment based on the 1910 census came from another mutation in apportionment methodology.

Congress abandoned the Quota Method and used a modified divisor method.

Modified Divisor Methods

Step 1. Select the House size, h.

Step 2. Apply a Basic Divisor Method to obtain h seats.

1910: $h = 433$ and Webster's method.

Any divisor between 711873 and 711882, inclusively, will work.

1920 Census

In the 1920 decade, for the only time in U. S. History, no census-based re-apportionment act was passed.

Congress could not agree on either the size of the House or on the method of apportionment. Further, the politics of prohibition played a significant role: the dries would not support any proposal that gave the wets more power.

Today

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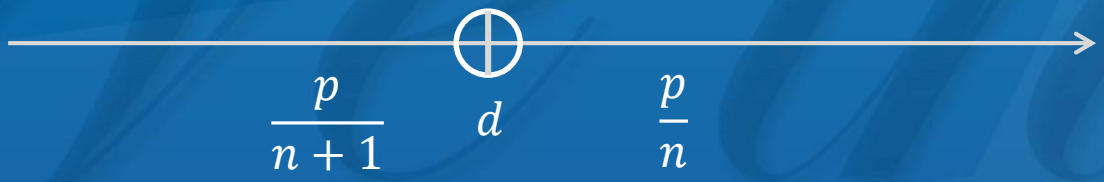

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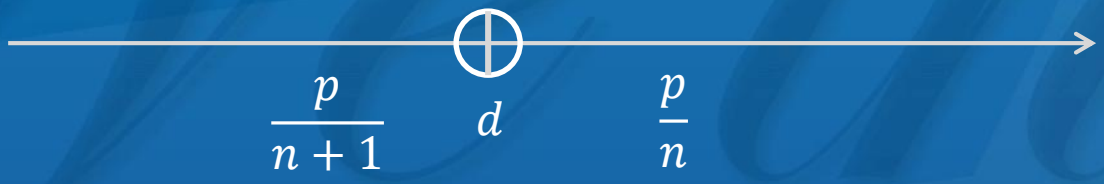

H-H:

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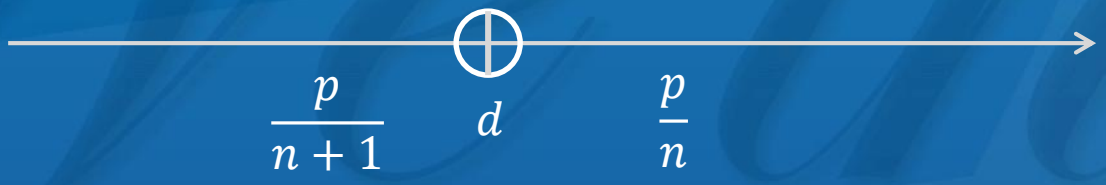

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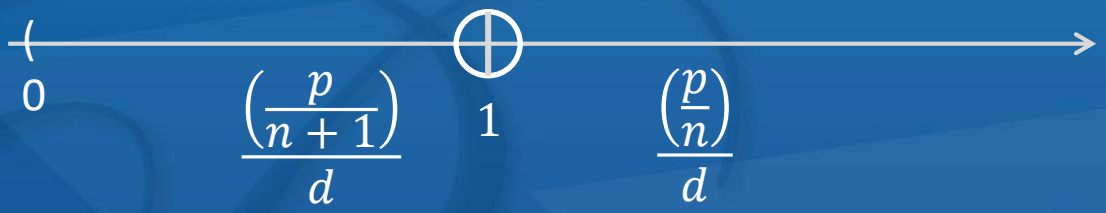

Criterion: $a = n+1$ iff $\frac{d}{\left(\frac{p}{n+1}\right)} \leq \frac{\left(\frac{p}{n}\right)}{d}$

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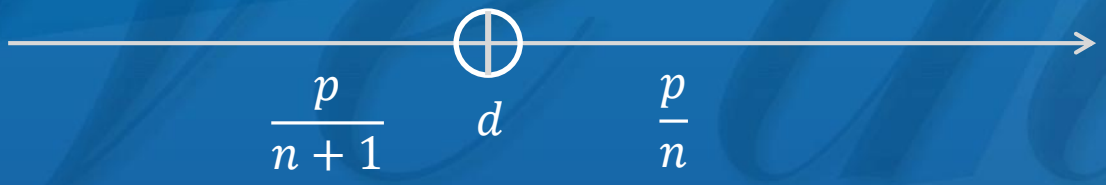

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There are no perfect apportionment methods.

Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

They Mean Well

A modified ***divisor*** method first fixes the House size, then seeks a divisor that when the state's quotients are rounded and summed, the house size is achieved.

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Webster: arithmetic mean $\frac{n+(n+1)}{2} = n + \frac{1}{2}$

Huntington-Hill: geometric mean $\sqrt{n(n+1)}$

Dean: harmonic mean $\frac{2}{\frac{1}{n}+\frac{1}{n+1}} = \frac{2n(n+1)}{2n+1}$

Montana

In the 1990 apportionment Montana lost one of its two seats it held for 80 years. In 1991 MT filed suit in federal district court (MT vs. US Dept Commerce).

MT argued the H-H method is unconstitutional and that either Dean's or Adams's method should be used. The federal judges voted 2-1 in favor of MT.

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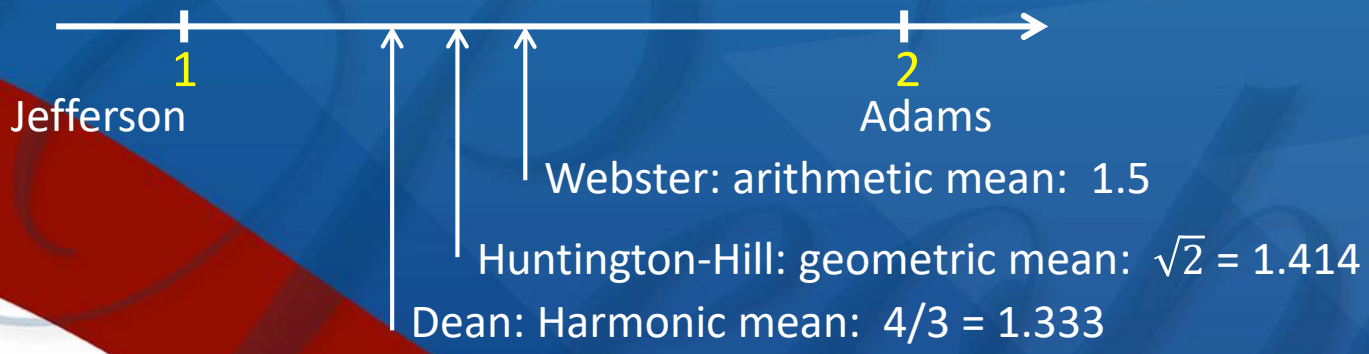

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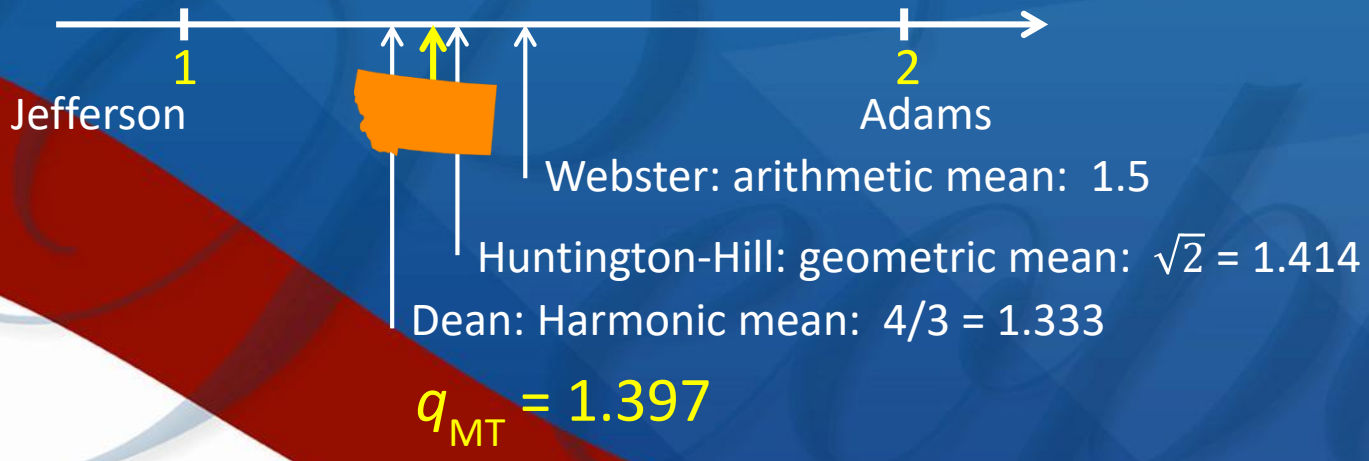

Today

Today the Census Bureau obtains apportionments using a priority technique of calculation rather than an ad-hoc technique of calculation.

Census Bureau video 2:09 minutes.
The Amazing Apportionment Machine

Apportionment by Priority

Step 1. Give one seat to each state.

Step 2. Attach a priority number to each state.

Step 3. Award seats one at a time by priority until the desired House size is reached.

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Priority number for a state with n seats

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$$\textit{Priority number for a state with } n \textit{ seats} = \frac{state\ population}{ave(n,n+1)}$$

Priority Numbers

Census 1790		
State	Population	Seats
Connecticut	236841	1
Delaware	55540	1
Georgia	70835	1
Kentucky	68705	1
Maryland	278514	1
Massachusetts	475327	1
New Hampshire	141822	1
New Jersey	179570	1
New York	331589	1
North Carolina	353523	1
Pennsylvania	432879	1
Rhode Island	68446	1
South Carolina	206236	1
Vermont	85533	1
Virginia	630560	1
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$$PN(1) = p/\sqrt{1 \times 2} = p/\sqrt{2}$$

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$$PN(4) = p/\sqrt{4 \times 5} = p/\sqrt{20}$$

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Census 1790			Priority Numbers H-H			
State	Population	Seats	1 seat	2 seats	3 seats	4 seats
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Vermont	85533	1	60480	34918	24691	19125
Virginia	630560	3			182026	140997
US	3615920	19				

Priority Numbers

16 VA 2
17 MA 2
18 PA 2
19 VA 3

Census 1790			Priority Numbers H-H			
State	Population	Seats	1 seat	2 seats	3 seats	4 seats
Connecticut	236841	1	167471	96689	68370	52959
Delaware	55540	1	39272	22674	16033	12419
Georgia	70835	1	50087	28918	20448	15839
Kentucky	68705	1	48581	28048	19833	15362
Maryland	278514	1	196939	113702	80400	62277
Massachusetts	475327	2		194051	137215	106286
New Hampshire	141822	1	100283	57898	40940	31712
New Jersey	179570	1	126975	73309	51837	40153
New York	331589	1	234468	135370	95721	74145
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Who's next???

Five Averages

	ave
• Greatest Divisors	max
• Harmonic Mean	HM
• Equal Proportions	GM
• Major Fractions	AM
• Smallest Divisors	min

The Last Seat

Who got the 435th seat?

	ave	435
• Greatest Divisors	max	IL
• Harmonic Mean	HM	MN
• Equal Proportions	GM	MN
• Major Fractions	AM	NC
• Smallest Divisors	min	WA

The Last Seat

Who gets the 436th seat?

	ave	435	436
• Greatest Divisors	max	IL	WA
• Harmonic Means	HM	MN	CA
• Equal Proportions	GM	MN	NC
• Major Fractions	AM	NC	MO
• Smallest Divisors	min	WA	PA

The Last Seat

Who gets the 436th seat?

	ave	435	436
• Greatest Divisors	max	IL	WA
• Harmonic Means	HM	MN	CA
• Equal Proportions	GM	MN	NC
• Major Fractions	AM	NC	MO
• Smallest Divisors	min	WA	PA

Priority list based on the 2010 census using the method of Equal Proportions.

The Future: Reform?

Four Proposals:

The Future: Reform?

Four Proposals:

- Thirty-thousand.org
- The Wyoming Rule
- Neubauer and Gartner
- Current method with rounding by Webster's Method.

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

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Jefferson basic divisor method.

CA: 1244 seats!

thirty-thousand.org

Here's an example of a concerned group:

http://www.thirty-thousand.org/

Thirty-thousand.org advocates 50000/representative.

This leads to a House with 6181 representatives using Webster's method of rounding.

California gets 747 seats.

The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).

http://en.wikipedia.org/wiki/Wyoming_Rule

http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/

The Wyoming Rule

Here are the results of applying the WY Rule to the 2000 and 2010 censuses.

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2000 smallest state: WY, 493782.

h = 569 Huntington-Hill

2010 smallest state: WY, 563626

h = 543 Dean HI

h = 542 Huntington-Hill

h = 540 Webster NJ, SD

A Proposal

A Proposal for Apportioning the House

Michael G. Neubauer, CSU Northridge, Mathematics
Margo G. (Gartner) Carr, Cerro Coso Community College

…the problem of finding a “good” house size and “right” apportionment method are best considered together.

Source: PSC 44(1), January 2011: 1—3.

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Source: PSC 44(1), January 2011: 1—3.

Webster's Method

The simplest reform would be to replace the geometric mean of decimal rounding in the Huntington-Hill method by the arithmetic mean of decimal rounding in Webster's method.

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The research of Balinski and Young has produced the following two key results. Since the Alabama paradox is a deal-breaker, then congressional apportionment must be based on a divisor method. Further, Webster's is the only rounding method that is unbiased towards either larger or smaller states.

nia977.wixsite.com/drbcap

Imported From Firefo | 2010 Census Apportio | U. S. Electoral College | One Touch Copy - Dv

Charles M. Biles, Ph.D.

Congressional Apportionment

HOME BIO NEWS & EVENTS RESOURCES CONTACT

Constitutional Congregressional Apportionment Problem

How many seats in the U. S. House of Representatives does each state get?

An answer is presented as an historical narrative with relevant and timely applications in an upcoming book, *The History of Congressional Apportionment.*

Chapter 1. Congressional Apportionment Based on the Census: 1790.
Chapter 2. Congressional Apportionment Based on the Census: 1800-1840.
Chapter 3. Congressional Apportionment Based on the Census: 1850-1890.
Chapter 4. Congressional Apportionment Based on the Census: 1900-1930.
Chapter 5. Congressional Apportionment Based on the Census: 1940-2010.
Chapter 6. An Historical Overture.

NEW

UNDER CONSTRUCTION

Charles Biles

The History of Congressional Apportionment

HSU Press

Last update: 13 June 2017.

Cover Graphic courtesy of The West Virginia Record

Thank You

It is time that I took my seat in this House!

http://www.nia977.wix.com/drbcap

Bonus Resources

Related Problems

Other problems related to apportionment include:

One Voter, One Vote: The Apportionment of Congressional Seats Reconsidered
Author(s): Howard A. Scarrow
Source: Polity, Vol. 22, No. 2 (Winter, 1989), pp. 253-268
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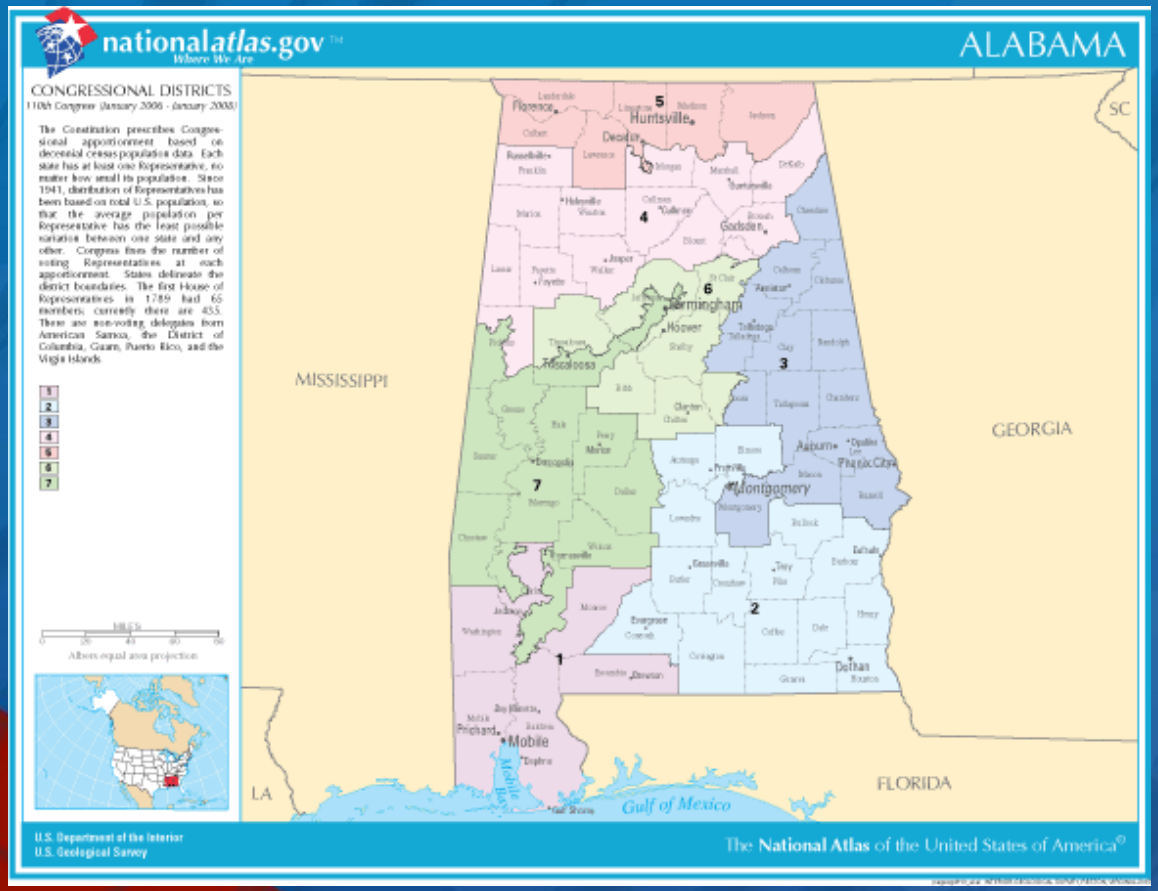
Gerrymandering



http://www.nationalatlas.gov/printable/congress.html#al

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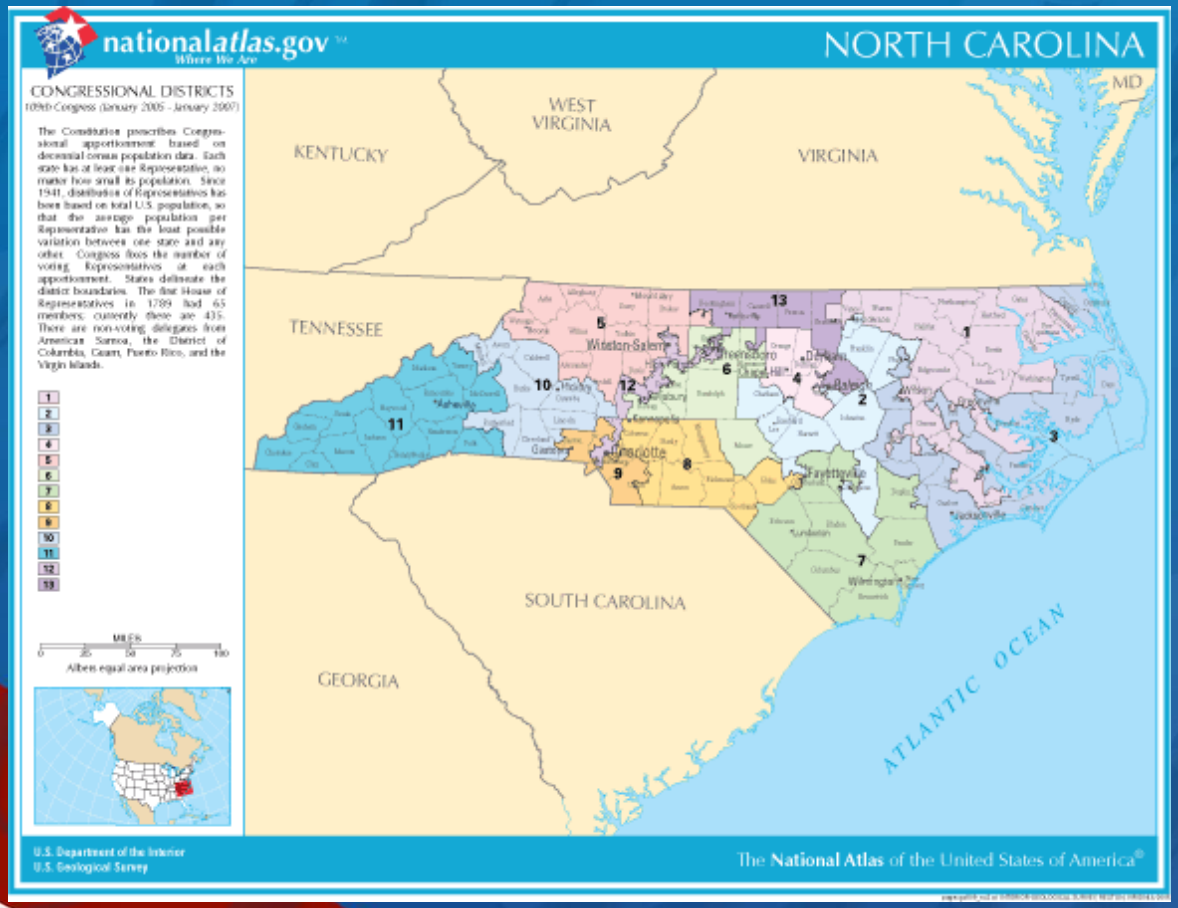

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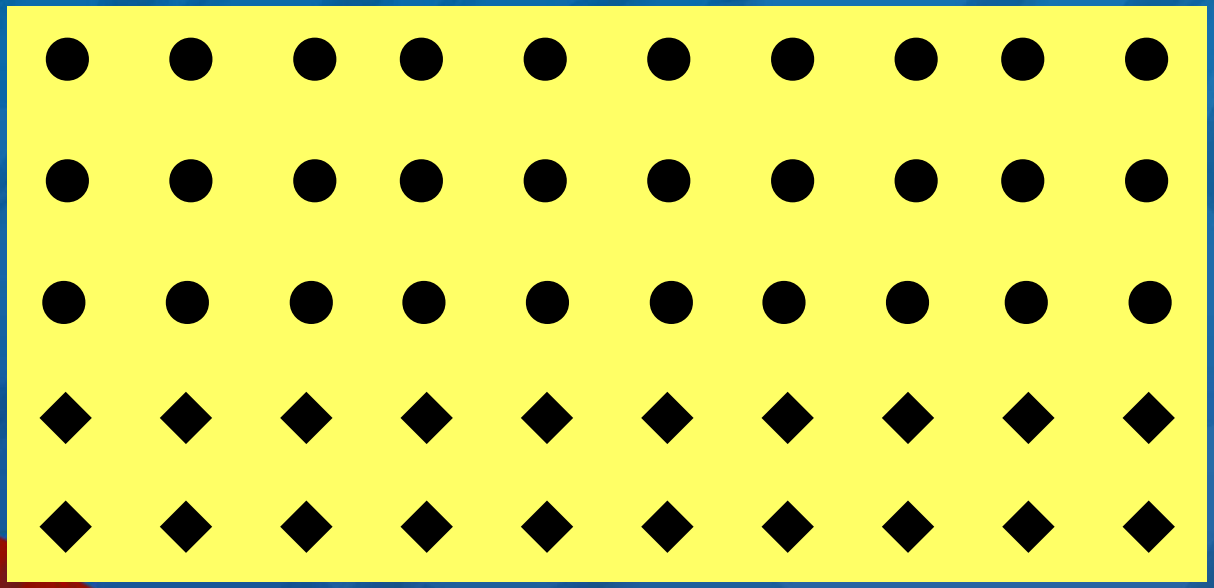
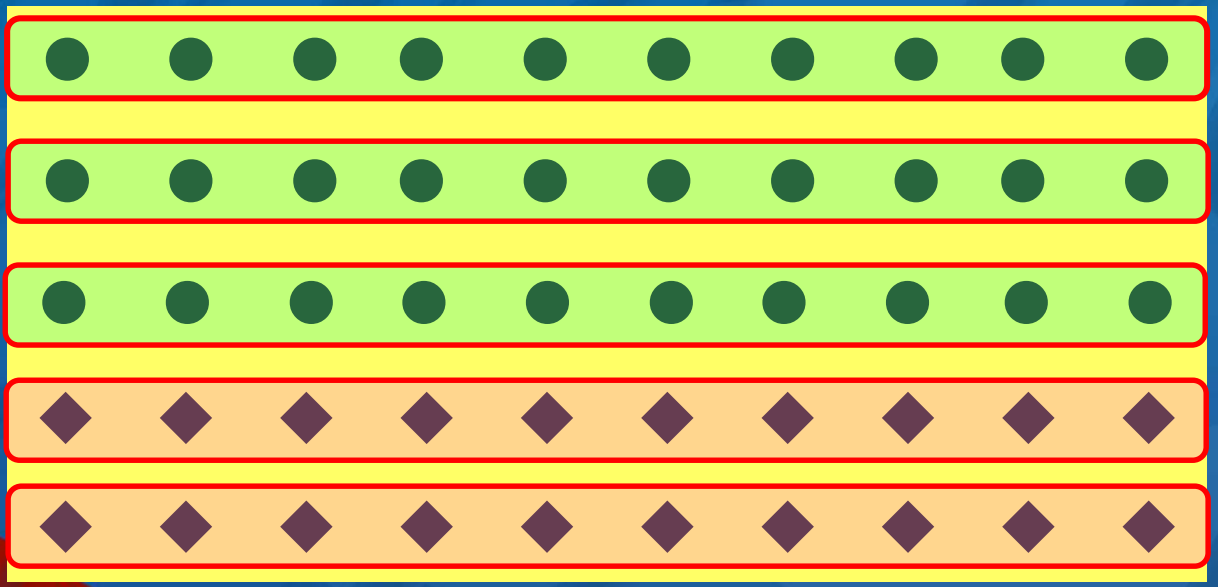


Apportion into five districts.

Gerrymandering



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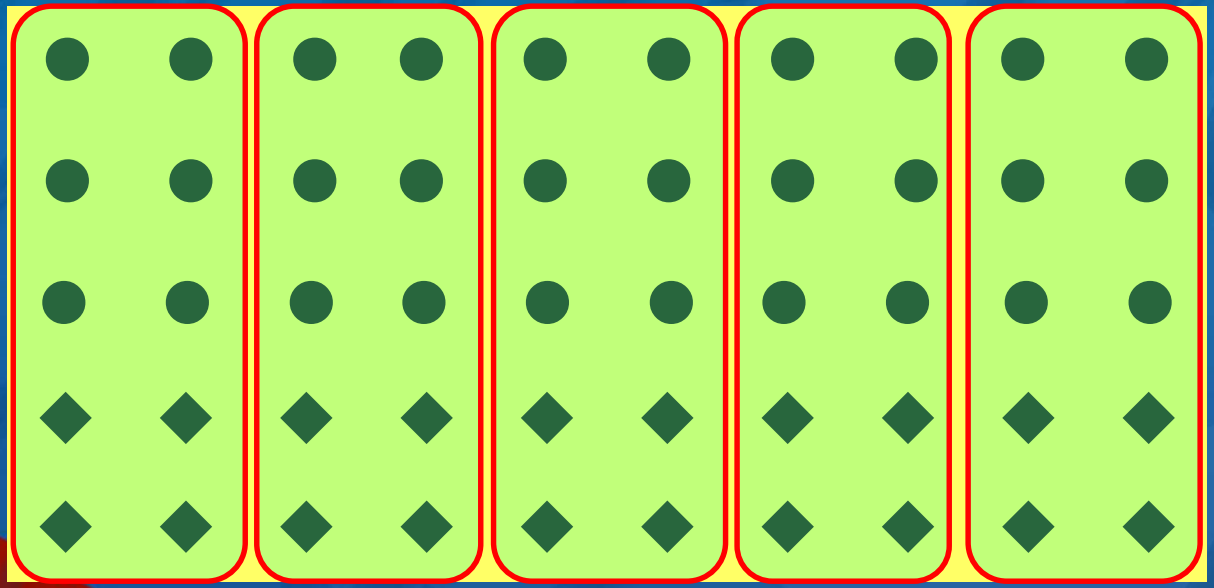

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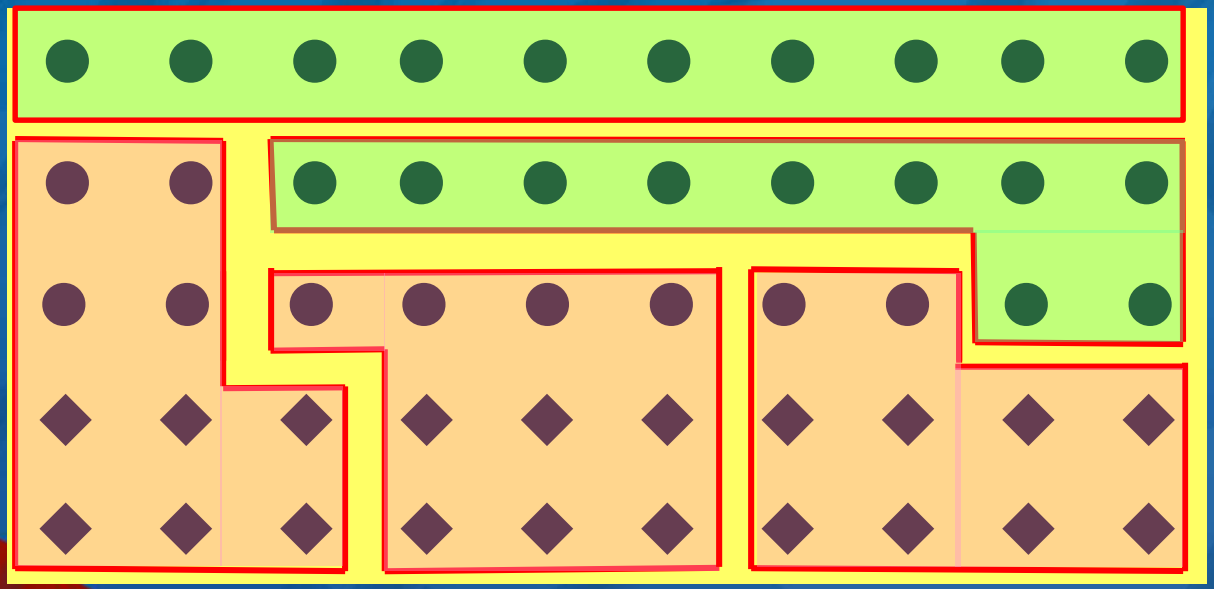

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US History

For any serious research of U.S. history, one must know about the Journals of Congress which includes the House Journal and the Senate Journal:

http://memory.loc.gov/ammem/amlaw/lwhj.html

Apportionment Problems

On appeal, the U.S. Supreme Court ruled unanimously that the H-H method was constitutional. The district court's decision was overturned.

http://www.law.cornell.edu/supct/html/91-860.ZS.html

http://caselaw.lp.findlaw.com/scripts/getcase.pl?court=US&vol=503&invol=442

Washington's Veto

United States [Philadelphia] April 5 1792.

Gentlemen of the House of Representatives

I have maturely considered the Act passed by the two Houses, intitled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.

First—The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.

Second—The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the seperate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.

George Washington.

Copy, DNA: RG 233, Second Congress, 1791–1793, Records of Legislative Proceedings, Journals; LB, DLC:GW. (from Philander Chase, et al., eds.,*The Papers of George Washington, Presidential Series, Vol. 10: March–August 1792* [Charlottesville, Va., 2002], 213-14).

First Apportionment Act

CHAP. XXIII.—*An Act for apportioning Representatives among the several States, according to the first enumeration.*

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the third day of March one thousand seven hundred and ninety-three, the House of Representatives shall be composed of members elected agreeably to a ratio of one member for every thirty-three thousand persons in each state, computed according to the rule prescribed by the constitution; that is to say: Within the state of New Hampshire, four; within the state of Massachussetts, fourteen; within the state of Vermont, two; within the state of Rhode Island, two; within the state of Connecticut, seven; within the state of New York, ten; within the state of New Jersey, five; within the state of Pennsylvania, thirteen; within the state of Delaware, one; within the state of Maryland, eight; within the state of Virginia, nineteen; within the state of Kentucky, two; within the state of North Carolina, ten; within the state of South Carolina, six; and within the state of Georgia, two members.

APPROVED, April 14, 1792.

1790: Why 33000?

State	Population	*d* =	30000	31000	32000	33000	34000	35000	36000	37000	38000	39000	40000
CN	236841		0.8947	0.6400	0.4013	0.1770	0.9659	0.7669	0.5789	0.4011	0.2327	0.0728	0.9210
DE	55540		0.8513	0.7916	0.7356	0.6830	0.6335	0.5869	0.5428	0.5011	0.4616	0.4241	0.3885
GA	70835		0.3612	0.2850	0.2136	0.1465	0.0834	0.0239	0.9676	0.9145	0.8641	0.8163	0.7709
KY	68705		0.2902	0.2163	0.1470	0.0820	0.0207	0.9630	0.9085	0.8569	0.8080	0.7617	0.7176
MD	278514		0.2838	0.9843	0.7036	0.4398	0.1916	0.9575	0.7365	0.5274	0.3293	0.1414	0.9629
MA	475327		0.8442	0.3331	0.8540	0.4038	0.9802	0.5808	0.2035	0.8467	0.5086	0.1879	0.8832
NH	141822		0.7274	0.5749	0.4319	0.2976	0.1712	0.0521	0.9395	0.8330	0.7322	0.6365	0.5456
NJ	179570		0.9857	0.7926	0.6116	0.4415	0.2815	0.1306	0.9881	0.8532	0.7255	0.6044	0.4893
NY	331589		0.0530	0.6964	0.3622	0.0482	0.7526	0.4740	0.2108	0.9619	0.7260	0.5023	0.2897
NC	353523		0.7841	0.4040	0.0476	0.7128	0.3977	0.1007	0.8201	0.5547	0.3032	0.0647	0.8381
PA	432879		0.4293	0.9638	0.5275	0.1175	0.7317	0.3680	0.0244	0.6994	0.3916	0.0995	0.8220
RI	68446		0.2815	0.2079	0.1389	0.0741	0.0131	0.9556	0.9013	0.8499	0.8012	0.7550	0.7112
SC	206236		0.8745	0.6528	0.4449	0.2496	0.0658	0.8925	0.7288	0.5739	0.4273	0.2881	0.1559
VT	85533		0.8511	0.7591	0.6729	0.5919	0.5157	0.4438	0.3759	0.3117	0.2509	0.1932	0.1383
VA	630560		0.0187	0.3406	0.7050	0.1079	0.5459	0.0160	0.5156	0.0422	0.5937	0.1682	0.7640
US	3615920		**8.5307**	**8.6426**	**6.9975**	**4.5733**	**6.3506**	**7.3120**	**9.4422**	**9.7276**	**8.1558**	**5.7159**	**9.3980**
		Unrepresented:	**255920**	**267920**	**223920**	**150920**	**215920**	**255920**	**339920**	**359920**	**309920**	**222920**	**375920**

Alabama Paradox

How is this possible?

State	House 299	House 300
AL	7.646	7.671
TX	9.640	9.672
IL	18.640	18.702

With the House size at 299, Alabama was the last state to be allotted an extra representative to make the House size because of it's decimal. When the House size was increased to 300, all states' quotas were increased by 0.33%. And there were two states that got the extra representatives; and, this time, Texas and Illinois beat out Alabama.

US Census Bureau

The U.S. Census Bureau is housed within the Department of Commerce.

Check out the U.S. Census Bureau for what it says about apportionment.

http://www.census.gov/

Summary 7-page brochure:
http://www.census.gov/prod/cen2010/briefs/c2010br-08.pdf

History of Legislation:
http://www.census.gov/history/www/reference/apportionment/apportionment_legislation_1790_-_1830.html

More!

For playing around, learning or teaching:

http://www.cut-the-knot.org/ctk/Democracy.shtml