## The Apportionment Controversy Bringing Down the House

Charles Biles, Ph.D.

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Website: http://www.nia977.wix.com/drbcap
"... no political problem is less susceptible of a precise solution than that which relates to the number most convenient for a representative legislature, ..."

## The Apportionment Question

How many seats in the U.S.
House of Representatives does each state get?

## CONGRESSIONAL SEATS


(US population: 309,183,463)/435 $=710,767$
http://www.census.gov/2010census/data/apportionment-data.php

## The Constitution: Article I

Section 1. All legislative Powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

## The Constitution: Article I

Section 2. The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, . . .

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The actual Enumeration shall be made within three Years after the first Meeting of the Congress . . ., and within every subsequent Term of ten Years, . . .

## The Constitution: Article I

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Representatives . . . shall be apportioned among the several States . . ., according to their respective Numbers, . . .

The actual Enumeration shall be made within three Years after the first Meeting of the Congress . . ., and within every subsequent Term of ten Years, . . .

The Number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at Least one Representative; . . .

## CONGRESSIONAL SEATS


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## The Apportionment Question: Two Views

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- Transformation View:
- Distribution View:


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- Transformation View: How to transform the census into seats in the House.
- Distribution View:


## The Apportionment Question: Two Views

- Transformation View: How to transform the census into seats in the House.
- Distribution View: How to distribute a fixed number of seats to the States.


## The Apportionment Question: Two Approaches

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- Constituency Approach:
- House Size Approach:


## The Apportionment Question: Two Approaches

- Constituency Approach: How many people should a congressperson represent?
- House Size Approach:


## The Apportionment Question: Two Approaches

- Constituency Approach: How many people should a congressperson represent?
- House Size Approach: How many seats should there be in the House?


## The First Census 1790

| State | Population |
| :---: | ---: |
| CT | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
| US | 3615920 |

The first apportionment population census.

## Source:

Balinski and Young, Fair Representation, Second Edition, 2001, page 158.

## The First Census 1790

| State |  | Population |
| :---: | ---: | ---: |
| CT | 5 | 236841 |
| DE | 1 | 55540 |
| GA | 3 | 70835 |
| KY |  | 68705 |
| MD | 6 | 278514 |
| MA | 8 | 475327 |
| NH | 3 | 141822 |
| NJ | 4 | 179570 |
| NY | 6 | 331589 |
| NC | 5 | 353523 |
| PA | 8 | 432879 |
| RI | 1 | 68446 |
| SC | 5 | 206236 |
| VT |  | 85533 |
| VA | 10 | 630560 |
| US | 65 | 3615920 |

The first apportionment population census.

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## The First Census 1790

| State |  | Population |
| ---: | ---: | ---: |
| CT | 5 | 236841 |
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| NH | 3 | 141822 |
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| NY | 6 | 331589 |
| NC | 5 | 353523 |
| PA | 8 | 432879 |
| RI | 1 | 68446 |
| SC | 5 | 206236 |
| VT | 2 | 85533 |
| VA | 10 | 630560 |
| US | 67 | 3615920 |

The first apportionment population census.

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Balinski and Young, Fair Representation, Second Edition, 2001, page 158.

## First Apportionment Bills

House Bill
State Population
CT 236841
DE 55540
GA 70835
KY 68705
MD 278514
MA 475327
NH 141822
NJ 179570
NY 331589
NC 353523
PA 432879
RI 68446
SC 206236
VT 85533
VA 630560

## First Apportionment Bills



## First Apportionment Bills



## First Apportionment Bills



## First Apportionment Bills

| Census |  | House Bill |  |
| :---: | :---: | :---: | :---: |
| State | Population | Divisor 3000 | Seats |
| CT | 236841 | 7.895 | 7 |
| DE | 55540 | 1.851 | 1 |
| GA | 70835 | 2.361 | 2 |
| KY | 68705 | 2.290 | 2 |
| MD | 278514 | 9.284 | 9 |
| MA | 475327 | 15.844 | 15 |
| NH | 141822 | 4.727 | 4 |
| NJ | 179570 | 5.986 | 5 |
| NY | 331589 | 11.053 | 11 |
| NC | 353523 | 11.784 | 11 |
| PA | 432879 | 14.429 | 14 |
| RI | 68446 | 2.282 | 2 |
| SC | 206236 | 6.875 | 6 |
| VT | 85533 | 2.851 | 2 |
| VA | 630560 | 21.019 | 21 |
|  |  |  | 112 |

## First Apportionment Bills

| Census |  |
| :---: | ---: |
| State | Population |
| CT | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
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| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
|  |  |

House Bill
Divisor 30000 Seats
7.8957
$1.851 \quad 1$
$2.361 \quad 2$
$2.290 \quad 2$
$9.284 \quad 9$
15.84415
$4.727 \quad 4$
$5.986 \quad 5$
$11.053 \quad 11$
$11.784 \quad 11$
$14.429 \quad 14$
$2.282 \quad 2$
$6.875 \quad 6$
$2.851 \quad 2$
$21.019 \quad 21$
112

Senate Bill
Divisor 33000 Seats

| 7.177 | 7 |
| ---: | ---: |
| 1.683 | 1 |
| 2.147 | 2 |
| 2.082 | 2 |
| 8.440 | 8 |
| 14.404 | 14 |
| 4.298 | 4 |
| 5.442 | 5 |
| 10.048 | 10 |
| 10.713 | 10 |
| 13.118 | 13 |
| 2.074 | 2 |
| 6.250 | 6 |
| 2.592 | 2 |
| 19.108 | 19 |
|  | 105 |

## Hamilton's Method

Federalists apply a new idea:

1. Determine the House size, $h$.
2. Calculate each state's fair share of $h$ :

$$
\text { quota }=h \times \frac{\text { state population }}{\text { US population }}
$$

## The House Bill

Census

| State |  |
| :---: | ---: |
| PTopulation | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
|  |  |

House Bill

## Divisor 30000 Seats

7.8957
$1.851 \quad 1$
$2.361 \quad 2$
$2.290 \quad 2$
$9.284 \quad 9$
15.84415
$4.727 \quad 4$
$5.986 \quad 5$
$11.053 \quad 11$
$11.784 \quad 11$
$14.429 \quad 14$
$2.282 \quad 2$
$6.875 \quad 6$
$2.851 \quad 2$
$21.019 \quad 21$

## The House Bill

Census

| State |  |
| :---: | ---: |
| CT | 2368 ation |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
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## House Bill

## Divisor 30000 Seats

7.8957
$1.851 \quad 1$
$2.361 \quad 2$
$2.290 \quad 2$
$9.284 \quad 9$
$15.844 \quad 15$
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$5.986 \quad 5$
$11.053 \quad 11$
$11.784 \quad 11$
$14.429 \quad 14$
$2.282 \quad 2$
$6.875 \quad 6$
$2.851 \quad 2$
$\begin{array}{rr}21.019 \quad 21 \\ & 112\end{array}$

## The House Bill

Census

| State |  |
| :---: | ---: |
| Copulation | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
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| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
|  |  |

House Bill

| Divisor $\mathbf{3 0 0 0 0}$ | Seats | Quota $\boldsymbol{h}=\mathbf{1 1 2}$ |
| :---: | :---: | ---: |
| 7.895 | 7 | 7.336 |
| 1.851 | 1 | 1.720 |
| 2.361 | 2 | 2.194 |
| 2.290 | 2 | 2.128 |
| 9.284 | 9 | 8.627 |
| 15.844 | 15 | 14.723 |
| 4.727 | 4 | 4.393 |
| 5.986 | 5 | 5.562 |
| 11.053 | 11 | 10.271 |
| 11.784 | 11 | 10.950 |
| 1.429 | 14 | 13.408 |
| 2.282 | 2 | 2.120 |
| 6.875 | 6 | 6.388 |
| 2.851 | 2 | 2.649 |
| 21.019 | 21 | 19.531 |
|  | 112 | 112 |

## The House Bill

Census

| State |  |
| :---: | ---: |
| Copulation | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
|  |  |

House Bill

| Divisor $\mathbf{3 0 0 0 0}$ | Seats | Quota $\boldsymbol{h}=\mathbf{1 1 2}$ |
| :---: | :---: | ---: |
| 7.895 | 7 | 7.336 |
| 1.851 | 1 | 1.720 |
| 2.361 | 2 | 2.194 |
| 2.290 | 2 | 2.128 |
| 9.284 | 9 | 8.627 |
| 15.844 | 15 | 14.723 |
| 4.727 | 4 | 4.393 |
| 5.986 | 5 | 5.562 |
| 11.053 | 11 | 10.271 |
| 11.784 | 11 | 10.950 |
| 1.429 | 14 | 13.408 |
| 2.282 | 2 | 2.120 |
| 6.875 | 6 | 6.388 |
| 2.851 | 2 | 2.649 |
| 21.019 | 21 | 19.531 |
|  | 112 | 112 |

The Quota Rule is violated.

## The Senate Bill

Census
Senate Bill

| State |  |
| :---: | ---: |
| CT | 236841 |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
|  |  |


| Divisor $\mathbf{3 3 0 0 0}$ Seats |  |
| :---: | ---: |
| 7.177 | 7 |
| 1.683 | 1 |
| 2.147 | 2 |
| 2.082 | 2 |
| 8.440 | 8 |
| 14.404 | 14 |
| 4.298 | 4 |
| 5.442 | 5 |
| 10.048 | 10 |
| 10.713 | 10 |
| 13.118 | 13 |
| 2.074 | 2 |
| 6.250 | 6 |
| 2.592 | 2 |
| 19.108 | 19 |
|  | 105 |

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## The Senate Bill

Census Senate Bill

| State |  |
| :---: | ---: |
| CT | 236841 a |
| DE | 55540 |
| GA | 70835 |
| KY | 68705 |
| MD | 278514 |
| MA | 475327 |
| NH | 141822 |
| NJ | 179570 |
| NY | 331589 |
| NC | 353523 |
| PA | 432879 |
| RI | 68446 |
| SC | 206236 |
| VT | 85533 |
| VA | 630560 |
|  |  |


| Divisor $\mathbf{3 3 0 0 0}$ Seats | Quota $\boldsymbol{h}=\mathbf{1 0 5}$ |  |
| :---: | :---: | :---: |
| 7.177 | 7 | 6.877 |
| 1.683 | 1 | 1.613 |
| 2.147 | 2 | 2.057 |
| 2.082 | 2 | 1.995 |
| 8.440 | 8 | 8.088 |
| 14.404 | 14 | 13.803 |
| 4.298 | 4 | 4.118 |
| 5.442 | 5 | 5.214 |
| 10.048 | 10 | 9.629 |
| 10.713 | 10 | 10.266 |
| 13.118 | 13 | 12.570 |
| 2.074 | 2 | 1.988 |
| 6.250 | 6 | 5.989 |
| 2.592 | 2 | 2.484 |
| 19.108 | 19 | 18.310 |
|  | 105 | 105 |
|  |  |  |

## Problem

Census Senate Bill

| State | Population | Divisor 33000 Seats |  | Quota $h=105$ |
| :---: | :---: | :---: | :---: | :---: |
| CT | 236841 | 7.177 | 7 | 6.877 |
| DE | 55540 | 1.683 | 1 | 1.613 |
| GA | 70835 | 2.147 | 2 | 2.057 |
| KY | 68705 | 2.082 | 2 | 1.995 |
| MD | 278514 | 8.440 | 8 | 8.088 |
| MA | 475327 | 14.404 | 14 | 13.803 |
| NH | 141822 | 4.298 | 4 | 4.118 |
| NJ | 179570 | 5.442 | 5 | 5.214 |
| NY | 331589 | 10.048 | 10 | 9.629 |
| NC | 353523 | 10.713 | 10 | 10.266 |
| PA | 432879 | 13.118 | 13 | 12.570 |
| RI | 68446 | 2.074 | 2 | 1.988 |
| SC | 206236 | 6.250 | 6 | 5.989 |
| VT | 85533 | 2.592 | 2 | 2.484 |
| VA | 630560 | 19.108 | 19 | 18.310 |
|  |  |  | 105 | 105 |

Large states are favored over small states.

## Hamilton's Method

| State | Population |  |
| :---: | :---: | :---: |
| CT | 236841 |  |
| DE | 55540 |  |
| GA | 70835 |  |
| KY | 68705 |  |
| MD | 278514 |  |
| MA | 475327 |  |
| NH | 141822 |  |
| NJ | 179570 |  |
| NY | 331589 |  |
| NC | 353523 |  |
| PA | 432879 |  |
| RI | 68446 |  |
| SC | 206236 |  |
| VT | 85533 |  |
| VA | 630560 |  |
| US | 3615920 | 120.5307 |
|  |  | = 30000 |

## Hamilton's Method

| State | Population | $h=120$ |
| :---: | :---: | :---: |
| CT | 236841 |  |
| DE | 55540 |  |
| GA | 70835 |  |
| KY | 68705 |  |
| MD | 278514 |  |
| MA | 475327 |  |
| NH | 141822 |  |
| NJ | 179570 |  |
| NY | 331589 |  |
| NC | 353523 |  |
| PA | 432879 |  |
| RI | 68446 |  |
| SC | 206236 |  |
| VT | 85533 |  |
| VA | 630560 |  |
| US | 3615920 | 120.5307 |
|  |  | = 30000 |

## Hamilton's Method

| State | Population | $h=120$ | Quota |
| :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 |
| DE | 55540 |  | 1.843 |
| GA | 70835 |  | 2.351 |
| KY | 68705 |  | 2.280 |
| MD | 278514 |  | 9.243 |
| MA | 475327 |  | 15.774 |
| NH | 141822 |  | 4.707 |
| NJ | 179570 |  | 5.959 |
| NY | 331589 |  | 11.004 |
| NC | 353523 |  | 11.732 |
| PA | 432879 |  | 14.366 |
| RI | 68446 |  | 2.271 |
| SC | 206236 |  | 6.844 |
| VT | 85533 |  | 2.839 |
| VA | 630560 |  | 20.926 |
| US | 3615920 | 120.5307 | 120 |
| $d=30000$ |  |  |  |

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q |
| :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 |
| DE | 55540 |  | 1.843 | 1 |
| GA | 70835 |  | 2.351 | 2 |
| KY | 68705 |  | 2.280 | 2 |
| MD | 278514 |  | 9.243 | 9 |
| MA | 475327 |  | 15.774 | 15 |
| NH | 141822 |  | 4.707 | 4 |
| NJ | 179570 |  | 5.959 | 5 |
| NY | 331589 |  | 11.004 | 11 |
| NC | 353523 |  | 11.732 | 11 |
| PA | 432879 |  | 14.366 | 14 |
| RI | 68446 |  | 2.271 | 2 |
| SC | 206236 |  | 6.844 | 6 |
| VT | 85533 |  | 2.839 | 2 |
| VA | 630560 |  | 20.926 | 20 |
| US | 3615920 | 120.5307 | 120 | 111 |
| $d=30000$ |  |  |  |  |

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |
| $d=30000$ |  |  |  |  |  |

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

First apportionment bill passed by Congress.

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

First apportionment bill passed by Congress.

26 March 1792: bill sent to President Washington for approval.

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

First apportionment bill passed by Congress.

26 March 1792: bill sent to President Washington for approval.

5 April 1792: Washington vetoes the bill.

## Hamilton's Method

| State | Population | $h=120$ | Quota | Lower Q | Appt |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CT | 236841 |  | 7.860 | 7 | 8 |
| DE | 55540 |  | 1.843 | 1 | 2 |
| GA | 70835 |  | 2.351 | 2 | 2 |
| KY | 68705 |  | 2.280 | 2 | 2 |
| MD | 278514 |  | 9.243 | 9 | 9 |
| MA | 475327 |  | 15.774 | 15 | 16 |
| NH | 141822 |  | 4.707 | 4 | 5 |
| NJ | 179570 |  | 5.959 | 5 | 6 |
| NY | 331589 |  | 11.004 | 11 | 11 |
| NC | 353523 |  | 11.732 | 11 | 12 |
| PA | 432879 |  | 14.366 | 14 | 14 |
| RI | 68446 |  | 2.271 | 2 | 2 |
| SC | 206236 |  | 6.844 | 6 | 7 |
| VT | 85533 |  | 2.839 | 2 | 3 |
| VA | 630560 |  | 20.926 | 20 | 21 |
| US | 3615920 | 120.5307 | 120 | 111 | 120 |

U.S.:
$3615920 / 120=30,132.66 \ldots$

## Hamilton's Method



## Basic Jefferson Method

After Washington's veto, in 6 days Congress passed the original Senate bill.

Washington signed it on 14 April 1972.

## Basic Jefferson Method

1. Decide on a divisor (constituency).
2. Calculate each state's quotient:
quotient = population/divisor
3. A state's apportion is the quotient rounded down.

## Basic Jefferson Method

1. Decide on a divisor (constituency).
2. Calculate each state's quotient:
quotient = population/divisor
3. A state's apportion is the quotient rounded down.

The House size is the sum of the state apportionments.

## First 50 years

The method was used until 1840.

* 1790: $s=15 ; d=33000 \Rightarrow h=105$
* 1800: $s=16 ; d=33000 \Rightarrow h=141$
* 1810: $s=17 ; d=35000 \Rightarrow h=181$
* 1820: $s=24 ; d=40000 \Rightarrow h=213$
* 1830: $s=24 ; d=47700 \Rightarrow h=240$


## Basic Jefferson Method

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- systematically favors larger states;
- can violate the Quota Rule.


## John Quincy Adams

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But Adams has flaws similar to Jefferson: it can violate the quota rule; systematically favors smaller states over larger states.

## James Dean

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Choose a divisor and calculate each state's quotient. Then round the decimal quotient that yields a constituency closest to the divisor.

## James Dean

Divisor: 50,000.
Vermont's population: 280,657.
Vermont's quotient: $280,657 / 50,000=5.613$.
Then, Jefferson assigns 5 seats to Vermont; Adams, 6 seats.
5 seats constituency: $280,657 / 5=56,131$.
6 seats constituency: $280,657 / 6=46,776$.
Now 46,776 is closer to the target of 50,000.
Dean awards Vermont 6 seats.

## Daniel Webster

Adams and Dean got Webster thinking.
Just round the decimal quotient normally: if (decimal part) < .5, then round down; if (decimal part) > .5, then round up.

## 1831

How to round a decimal?
Jefferson: down.
Adams: up.
Dean: closest to constituency.
Webster: normally.

## 1842

In 1842 the apportionment debate began with the political game: Divisor! On one day in the 242 member House, 59 motions were made to establish a divisor; values ranged from 30000 to 141000 .

## 1842

The Apportionment Act of 1842 specified the divisor 70680 (result: House of 223 ) with rounding using Webster's method.

## The Vinton Act

The Vinton Act of 1850 was passed to head off politicizing the census figures and adopt a permanent appropriation act.

Representative Samuel Vinton Whig, Ohio


## The Vinton Act

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Experience exposed problems with the Vinton Act.

## 1920

$>$ No re-apportionment act was passed.
$>$ Congress could not agree on the method of apportionment.
$>$ Prohibition played a significant role: the dries would not consider any allocation giving the wets more power.

## Lessons from History

Experience using the quota method shows that it is subject to counter-intuitive paradoxes; especially, the Alabama Paradox:

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Experience using the quota method shows that it is subject to counter-intuitive paradoxes; especially, the Alabama Paradox:
when the number of House seats is increased, a state's apportion may decrease.

## Alabama Paradox

Results from the 1900 census doomed Hamilton's method. In particular, Maine oscillated as follows:

3 members for House size 350-382, 386, 389-390
4 members for House size 383-385, 387-388, 391-400

## Today

The method used today is described in Public Law 291. It consists of a 1929 statute that freezes the House size (435) and a 1941 amendment that specifies the apportionment method of

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Suppose a state's quotient (state population/divisor) = q = n.d.

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Suppose a state's quotient (state population/divisor) = q = n.d.

Webster: round up if $q>n .5$.
Huntington-Hill: round up if $q \geq \sqrt{n(n+1)}$.

## The Aftermath

Michel Balinski, Professor of Mathematics at SUNY Stony Brook, and H. Peyton Young, Professor of Mathematics at Johns Hopkins University, proved the following theorem in 1982:

There are no perfect apportionment methods. Any method that satisfies the quota rule produces paradoxes; any method that is free of the Alabama paradox may violate the quota rule.

## The Aftermath

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In particular, any divisor method may violate the quota rule; any quota method produces paradoxes.

## Well-Rounded Ideas

A modified divisor method first fixes the House size, then seeks a divisor that when the state's quotients are rounded and summed, the house size is achieved.

## They Mean Well

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Jefferson

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## Apportionment Problems

In the 1990 apportionment, Montana lost one of its two seats it held for 80 years. In 1991 MT filed suit in federal district court (MT vs. US Dept Commerce).

Montana argued the H-H method is unconstitutional and that either Dean's or Adam's method should be used. The federal judges voted 2-1 in favor of Montana.


Webster: arithmetic mean
Huntington-Hill: geometric mean
Dean: harmonic mean

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## The Apportionment Question

How many seats in the U.S.
House of Representatives does each state get?

## Reform

Three Proposals:

- Thirty-thousand.org
- The Wyoming Rule
- Neubauer and Gartner


## thirty-thousand.org

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Comment: This leads to a House with 10283 representatives.

In 2010 Santa Rosa's population was 167815. So, Santa Rosa would have 5 seats in the House.

CA: 1244 seats!

## The Wyoming Rule

The Wyoming Rule is a basic divisor method in which the divisor is the population of the least populous state (currently WY; hence, the name).
http://en.wikipedia.org/wiki/Wyoming Rule
http://www.outsidethebeltway.com/representation-in-the-house-the-wyoming-rule/

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2000: smallest state: WY, 493782.
$h=569$ Huntington-Hill
2010: smallest state: WY, 563626

$$
h=543 \text { Dean } \mathrm{HI}
$$

$h=542$ Huntington-Hill
$h=540$ Webster NJ, SD

## A Proposal

A Proposal for Apportioning the House
Michael G. Neubauer, CSU Northridge, Mathematics
Margo G. Gartner (master's degree student)
...the problem of finding a "good" house size and "right" apportionment method are best considered together.

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## Thank You

It is time that I took my seat in this House!

For more: http://nia977.wix.com/drbcap

